BHARAT INSTITUTE OF ENGGINEERING AND TECHNOLOGY

(Approved by AICTE Affiliated to SCTE & VT)

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LECTURE NOTES

ON

STRUCTURAL DESIGN-I

CIVIL, 4th SEMESTER

PREPARED BY

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DEPARTMENT OF CIVIL ENGINEERING

30th Oct,
Thursday

REINFORCED CEMENT CONCRETE

CONCRETE
- → Cement (Binder)
- → Coarse aggregate (Strength)
- → Fine aggregate (void filler)
- → Water (hydration + workability).

STEEL
- → Mild Steel → Fe 250
- → HYSD
  - → Fe 415
  - → Fe 500

MS PLAIN,
only 6mm ϕ available

HYSD (ribbed or corrugated)

min 8mm, 10 mm, 12mm, 16mm, 20, 25, 32, 36 mm.

$$\boxed{\begin{aligned} E_s &= 200 \text{ GPa} \\ \alpha_s &= 12 \times 10^{-6}/^\circ C \\ \mu_s &= 0.3 \\ \gamma_s &= 7850 \text{ kg/m}^3 \end{aligned}}$$

* Length of bar:

$l = 12$ m : bars upto 25ϕ

$l = 6$ m : bars greater than 25ϕ

Nowadays, High Yield Strength Deformed (HYSD) are replaced by Thermo Mechanically Treated (TMT) steel bars. Chemical composition is similar in TMT & HYSD bars.

CORE
Ductile & tough.
(SOFT)

Hard, resistance to weathering

Mould containing molten steel is immersed in cold water which results in a soft core & hard outer surface.

→ Methods of RCC Design :

1. Working Stress Method.
2. Ultimate Load Method.
3. Limit State Method

* Working Stress Method. (Elastic Method (or) Modular Ratio Method (or) Factor of Safety Method)

– Design load = working (or) service load

– Design stresses / allowable stresses / permissible stresses

$$= \frac{\text{Characteristic strength of Material}}{\text{Factor of Safety}}$$

M20 ⇒ $f_{ck} = 20$ MPa ; Fe 250 ⇒ $f_y = 250$ MPa.

* Factor of Safety in 'Beams' :-

$\boxed{\text{Concrete} = 3}$ $\boxed{\text{Steel} = 1.78}$

- In WSM, design load is based on uniqueness theorem, design strength of material is based on lower bound theorem

- In this method, serviceability is not considered (for such a low level loads, serviceability is not required.

* Ultimate Load Method.

- Design load = Ultimate / Collapse load
  = Working load (or) Service load * Load factor.

- Load factor depends on load combinations.

- Design stresses / allowable / permissible stresses

$$= \frac{\text{Characteristic strength of Material}}{\text{Material Safety Factor}}$$

- Material Safety Factor 'in beams'

Concrete = 1.5
Steel = 1.15

- In ultimate load method,

Design load → based on upper bound theorem

Strength of material → based on lower bound theorem.

In this method, serviceability is not considered. ∴ it has not become popular.

* Limit State Method.

'Limit State' is a condition just before collapse. Upto this condition, member is safe to resist external loads and also gives proper service throughout its life.

Limit state method is divided into two :-

(i) Limit State of Collapse

a) Flexure (Bending) → beams.

b) Shear → beams

c) Compression → columns.

d) Torsion → End or L beams, beams curved in plan.

(ii) Limit State of Serviceability

a) Deflections

b) Cracking

c) Vibrations

d) Fire resistance.

e) Durability

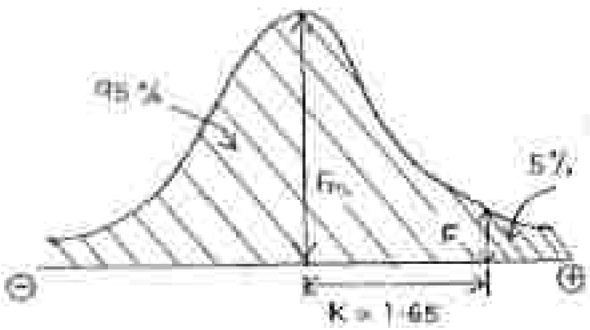
— Design load / Ultimate / Factored / collapse / Limiting load.

$$F_D = \text{Characteristic load} * \gamma_f$$

ie $\boxed{F_D = F \cdot \gamma_f}$

γ_f → partial safety factor for forces/loads (IS 875 - P(5))

— Characteristic load :- the load which has 95% probability of not being exceeded in the life of a structure is characteristic load



Fm = average / mean load.

$F = F_m + K(S)$

$\Rightarrow \boxed{F = F_m + 1.65\,(S)}$

F → characteristic load.

$$F_m = \frac{F_1 + F_2 + F_3 + \cdots}{n}$$

$$\text{Standard deviation, } S = \sqrt{\frac{(F_m - F_i)^2}{(n-1)}}$$

NOTE :

- RCC limit state is based on 95% probability load.
- Pavement design is based on 98% probability load.

Q. The live loads acting over a structure at different conditions are given below. Determine characteristic load 30 kN/m, 15 kN/m, 20 kN/m, 45 kN/m, 50 kN/m.

$$F_m = \frac{30 + 15 + 20 + 45 + 50}{5} = 32 \text{ kN/m}$$

- As per IS 456, to have characteristic values, min 30 sample are required to have proper corelation.

$$S = \sqrt{\frac{(32-30)^2 + (32-15)^2 + (32-20)^2 + (32-45)^2 + (32-50)^2}{5-1}}$$

$= \underline{15.24}$

Characteristic load, $F = F_m + KS = 32 + 1.65 \times 15.25$

$= \underline{57.16}$ kN/m.

- In case of EL (Earthquake Load), replaced WL (Wind Load) with EL in the combinations for partial safety factor, γ_f.

- Both WL & EL occuring critically together is impossible is why they are not considered together

Q. Calculate design load in collapse and serviceability seperately for :

DL = 150 kN/m ; LL = 250 kN/m ; $\underbrace{WL = 25 \text{ kN/m}; \; EL = 32 \text{ kN/m}}_{\text{use max of EL \& WL.}}$

- Design load for collapse :

(i) $1.5\,DL + 1.5\,LL = 1.5(150 + 250)$
$= 600$ kN/m

(ii) $1.5\,DL + 1.5\,EL = 1.5(150 + 32)$
$= 273$ kN/m

(iii) $1.2\,DL + 1.2\,LL + 1.2\,EL = 1.2(150 + 250 + 32)$
$= 518$ kN/m

Use max. of three values = $\underline{\underline{600 \text{ kN/m}}}$

- Design load for Service :

(i) $1.0\,DL + 1.0\,LL = 150 + 250 = \underline{\underline{400}}$

(ii) $1.0\,DL + 1.0\,EL = 150 + 32 = \underline{\underline{182}}$

(iii) $1\,DL + 0.8\,LL + 0.8\,EL = 150 + 0.8(250 + 32)$
$= \underline{\underline{375.6}}$

Service design load = $\underline{\underline{400 \text{ kN/m}}}$

- Design Strength of Material / Design Stresses / Allowable / Permissible Stresses :

$$\boxed{f_d = \frac{f}{\gamma_m}}$$

$f \rightarrow$ characteristic strength of material.

$\gamma_m \rightarrow$ partial safety factor for material.

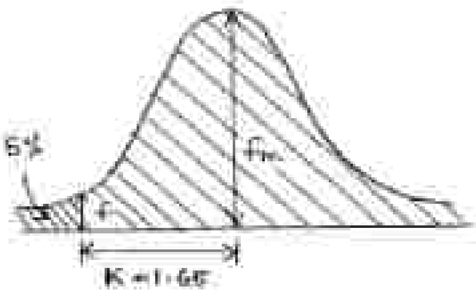
Characteristic strength of material :-

The strength below which not more than 5% of test results are expected to fall.

$f < f_m$.

$$\boxed{f = f_m - KS}$$

where $f_m \rightarrow$ mean / average strength



Design load > Average

Design Strength < Average.

Material	Collapse	Service
Concrete	1.5	1.0
Steel	1.15	1.0

Q. The compressive strength of standard 15 cm cubes are given below. Determine characteristic strength of concrete.

28 MPa, 33 MPa, 30 MPa, 18 MPa, 40 MPa.

$$f_m = \frac{28+33+30+18+40}{5} = \frac{149}{5} = \underline{\underline{29.8}}\ MPa$$

$$S = \sqrt{\frac{\Sigma (f_m - f_i)^2}{n-1}} = \sqrt{\frac{(29.8-28)^2+(29.8-33)^2+(29.8-30)^2+\ldots}{4}}$$

$$= \underline{\underline{8.03}}$$

$$f = f_m - KS = 29.8 - 1.65 \times 8.03$$

$$= \underline{\underline{16.557}}\ MPa$$

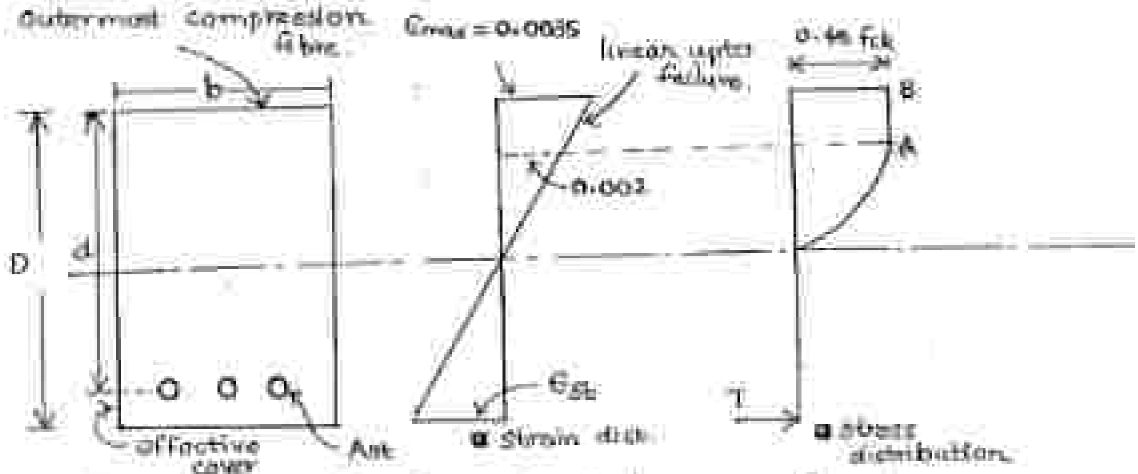
03. Singly Reinforced Sections

(LIMIT STATE METHOD)

→ Assumptions:

1. Euler - Bernoullie

As per Bernoulli, there is no distortion in the shape of c/s. Strain distribution is linear as per Bernoulli, with zero strain at neutral axis and max at extreme fibres. Bernoulli's assumption is valid for composite members also, valid upto failure ∴ it can be used in Limit State method also.

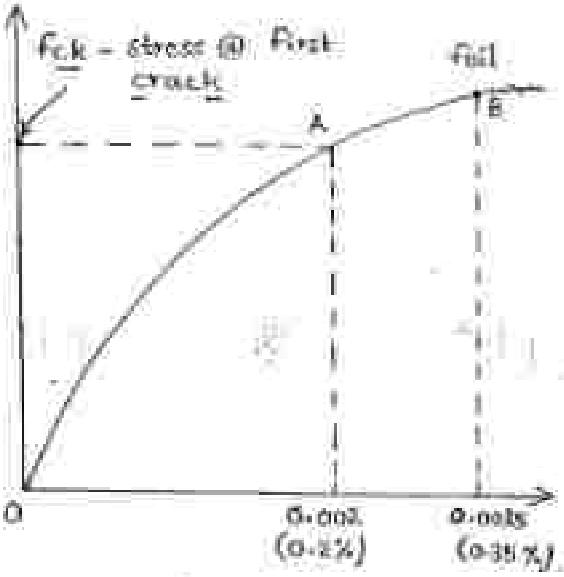


∴ LSM is a strain oriented approach. (WSM is stress oriented approach)

⊙ Modulus of rupture, $\boxed{f_{cr} = 0.7\sqrt{f_{ck}}}$

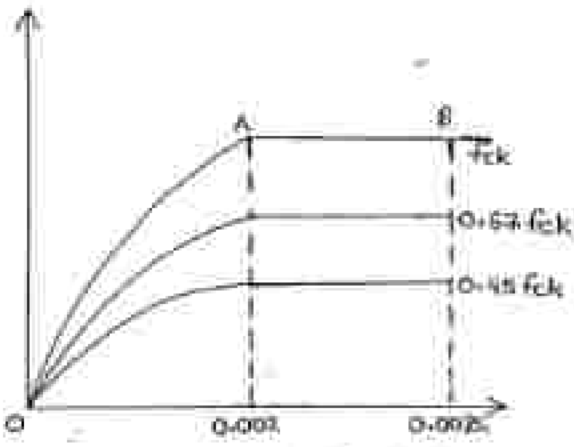
f_{cr} is tensile strength of concrete in bending / flexure and is calculated by two point load on beam / prism

2. In original $\sigma - \epsilon$ curve, OA is 'nonlinear elastic zone' and AB is 'crack widening zone'. In the revised stress-strain curve, crack widening zone is made flat and treated as 'plastic zone'.



■ Original $\sigma - \epsilon$ curve of concrete.

For characteristic compressive strength, min 30 samples are to be considered.
It is not possible to test 30 samples all the time. ∴ for random value to compressive strength, 3 cubes average @ 28 days with variation should not be more than ±15% should be used.



■ Revised as per IS 456-2000

• fck is cube compressive strength.
The strength of concrete in a structural member will be 33% less than cube strength due to size and slenderness effect.

$$\text{Design compressive strength of concrete} = \frac{0.67\ f_{ck}}{\gamma_m}$$

$\gamma_m \rightarrow$ partial safety safety factor for concrete (= 1.5).

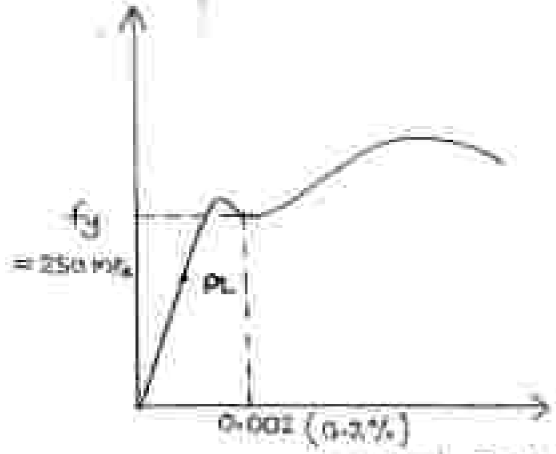
$$\boxed{\text{Design compressive strength of concrete} = 0.45\ f_{ck}}$$

∴ overall FOS used for concrete is 2.22 $\left(= \frac{1}{0.45}\right)$

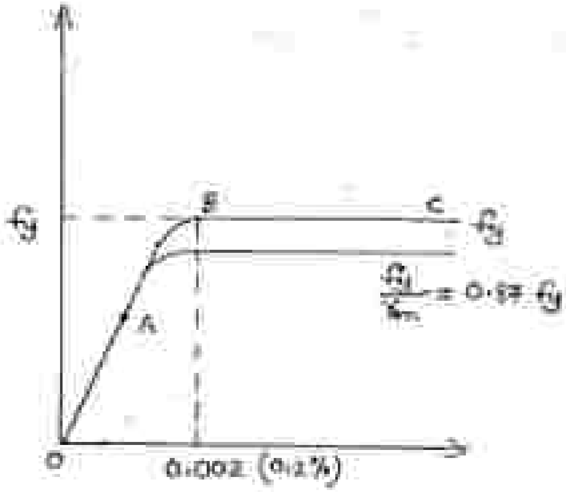
⊙ Crack occurs at $\epsilon = 0.002 = \frac{\delta l}{l}$

3. Stress distribution curve is parabolic and rectangular

4.



▪ original σ – ε curve for MS



$\frac{f_y}{\gamma_m} = 0.87 f_y$

▪ Revised (idealized),

OA → linear elastic

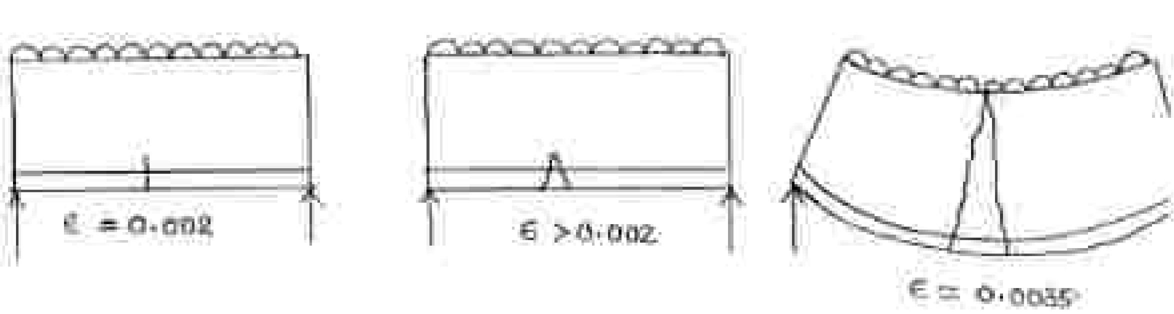
AB → non linear elastic

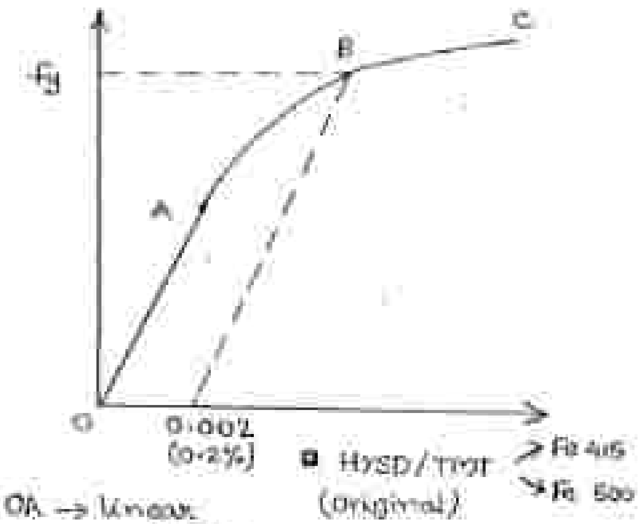
BC → plastic.

Design strength of steel is $= \frac{f_y}{\gamma_m} = \frac{f_y}{1.15} = 0.87 f_y$.

$$\boxed{\text{Design strength of steel is } 0.87 f_y}$$

Strain at failure for steel is not given as the concrete fails much earlier to the failure of steel.

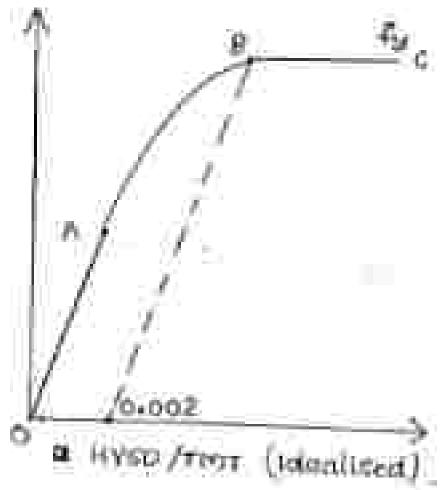




OA → linear elastic

AB → non linear elastic

BC → strain hardening



OA → linear elastic

AB → non linear elastic

BC → plastic

- All the stress-strain curves in LSM are considered to be elasto plastic (or) visco elastic.

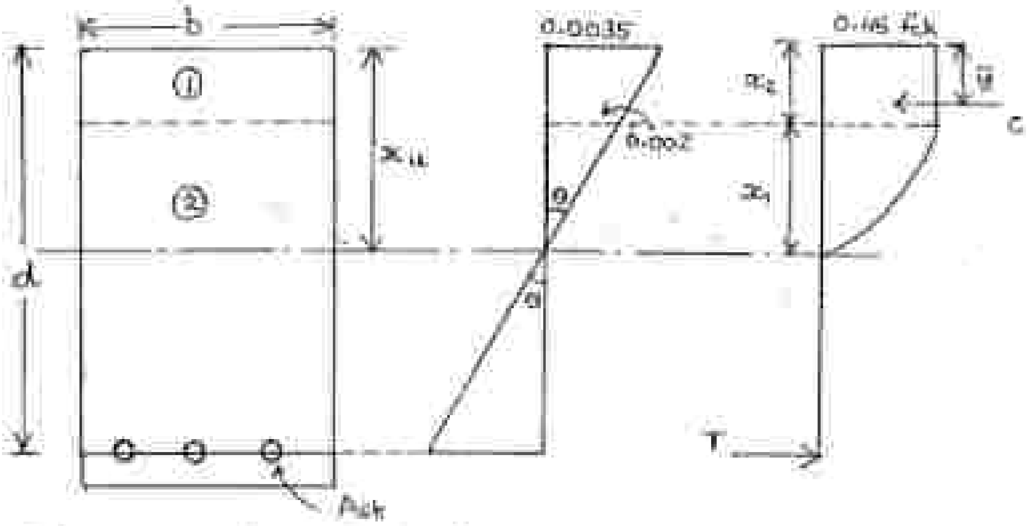
Strain in steel }. $\epsilon_{st} \nless 0.002 + \frac{f_y}{\gamma_m E_s}$

$$\Rightarrow \boxed{\epsilon_{st} \geq 0.002 + \frac{0.87 f_y}{E_s}}$$

- To have proper indications of failure with cracks in the tension zone, the strain in the steel at failure should be more than $\left(0.002 + \frac{0.87 f_y}{E_s}\right)$.

- If strain in steel is less than 0.002, there will be sudden failure in concrete in the compression zone without indications.

→ Stress Block Parameters



From similar triangles of strains,

$$\tan\theta = \frac{0.0035}{x_u} = \frac{0.002}{x_1}$$

$$\Rightarrow \boxed{x_1 = \frac{4}{7}x_u = 0.57\,x_u}$$

$$\boxed{x_2 = \frac{3}{7}x_u = 0.43\,x_u}$$

* Tensile force, T = design stress in steel * A_{st}.

$$\boxed{T = 0.87 f_y A_{st}}$$

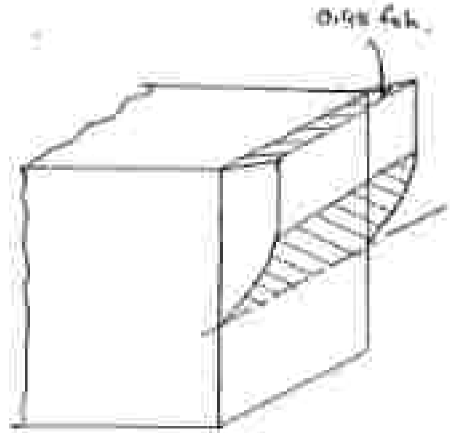
* Compressive force, $C = C_1 + C_2$

$C_1 \rightarrow$ average stress on ① * A_1

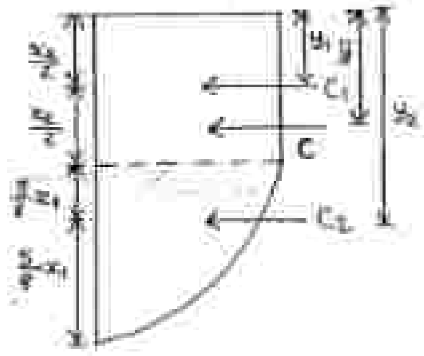
$C_2 \rightarrow$ average stress on ② * A_2

$$C_1 = (0.45 f_{ck}) \times (b x_2)$$

$$C_2 = \frac{2}{3}(0.45 f_{ck} + 0) * (b x_1)$$



$$\boxed{C = 0.36\, f_{ck}\, (b\, x_u)}$$



$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

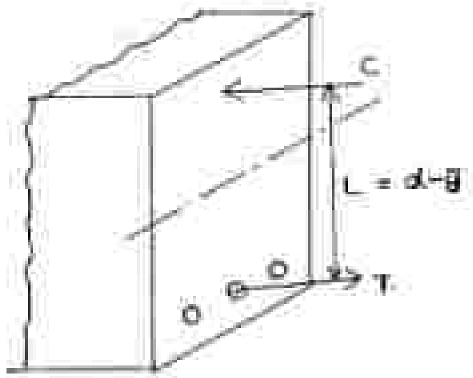
$$= \frac{\left(\frac{2}{3}\, 0.45\, f_{ck}\, (b x_1) \times \left(\frac{x_2}{2} + \frac{3}{8} x_1\right) + 0.45\, f_{ck}\, (b x_2) \times \frac{x_2}{2}\right)}{0.36\, f_{ck}\, b\, x_u}$$

$$\Rightarrow \boxed{\bar{y} = 0.42\, x_u}$$

Under all circumstances till failure, C = T

→ Moment of Resistance.

Resistance of cross section for externally applied moment due to loads.



* Based on Compressive force

$$M_u = CL$$

$$= 0.36\, f_{ck}\, b x_u\, (d - 0.42\, x_u) \rightarrow (1)$$

* Based on tensile force

$$M_u = TL$$

$$= 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u) \rightarrow (2)$$

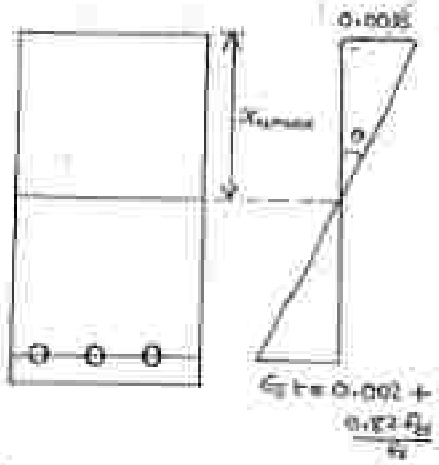
For beam to be stable, $(M_u)_C = (M_u)_T$

→ Neutral Axis

(i) Balanced / Limiting / Critical / Maximum NA (x_{umax})

This is the NA for balanced failure where both steel and concrete fails at the same time.

$$\tan\theta = \frac{0.0035}{x_{umax}} = \frac{\epsilon_{st}}{d - x_{umax}}$$



where $\epsilon_{st} = 0.002 + \frac{0.87 f_y}{E_s}$

∴ x_{umax} depends only on grade of steel; independent of grade of concrete (as for all grades of concrete, max strain is 0.0035)

$$\Rightarrow \frac{0.0035}{x_{umax}} = \frac{0.002 + \frac{0.87 \times 250}{2 \times 10^5}}{d - x_{umax}} \quad \{\text{for Fe 250}\}$$

$$\boxed{\frac{x_{umax}}{d} = 0.53 \ ; \ \text{Fe } 250}$$

$$\boxed{\frac{x_{umax}}{d} = 0.48 \ ; \ \text{Fe } 415}$$

$$\boxed{\frac{x_{umax}}{d} = 0.46 \ ; \ \text{Fe } 500}$$

(ii) Actual NA / NA of beam at a given condition (x_u).

$$C = T$$

$$0.36 f_{ck} \, b x_u = 0.87 f_y A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \, b}}$$

→ Balanced Section

- used in design of a member.
- In this section, both C & T reach to maximum values and fail at the same time. This is theoretically possible and practically impossible.

$$\boxed{x_{u,max} = x_u}$$

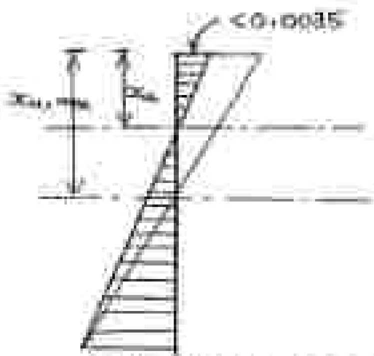
⊙ Moment of resistance, $M_{u,limit}$ = eqn ① or ② with $x_{u,max}$

- even in a balanced section, concrete fails suddenly.

→ Under Reinforced Section.

- reinforcement is less than that in a balanced section.
- In this section, steel first reaches to the maximum value, at that point concrete is having less than the maximum values of strain and stress. There will be lot of indications before failure. Ultimately concrete fails in a gradual manner with the support of steel.

$$\boxed{x_u < x_{u,max}}$$

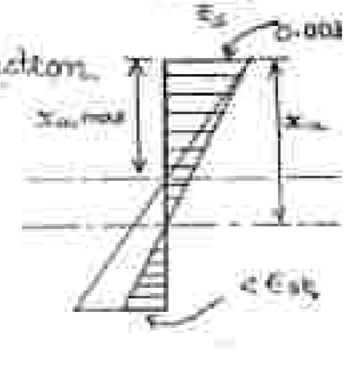


⊙ MR, M_u = eqn ① or ② with x_u

$\epsilon_{st} = 0.002 + \frac{0.87 f_y}{E_s}$

→ Over Reinforced Section.

- reinforcement is more than balanced section.
- concrete fails suddenly. DANGER !!



$$\boxed{x_u > x_{u,max}}$$

- In case of design, ORS should be strictly prohibited as per IS 456. In case of an existing beam found to be over reinforced, reduce its load carrying capacity or moment carrying capacity to that of a balanced section using equation ① only. with x_{umax}.

$$\boxed{M_{u,limit} = 0.36 f_{ck}\, b\, x_{umax} (d - 0.42\, x_{umax})}$$

→ Limiting % steel (Steel required for a Balanced Section)

$$C = T$$

$$0.36 f_{ck}\, b.\, x_{umax} = 0.87 f_y A_{st}$$

$$\boxed{P_{bal} = 100 \frac{A_{st}}{bd} = \frac{0.36 f_{ck}}{0.87 f_y} \left(\frac{x_{u\,max}}{d}\right) \times 100}$$

Eg: M20 → $f_{ck} = 20$ MPa

Fe415 → $f_y = 415$ MPa

$$P_{bal} = \frac{0.36 \times 20}{0.87 \times 415} \times (0.48) \times 100 = 0.957\%$$

0.957% is the % steel required for a balanced section

So provide < 0.951% for under reinforced section ($\simeq 0.9\%$).

→ Limiting or Balanced Moment of Resistance

$$M_{u\,limit} = \text{eqn ① with } x_{umax}$$

Eg: MS (Fe 250) → $x_{umax} = 0.53d$

$$M_{u,limit} = 0.36 f_{ck}\, b\, x_{u\,max} (d - 0.42\, x_{u\,max}).$$

$$\boxed{M_{u,limit} = 0.148\ f_{ck}\ bd^2}$$

Eg: HYSD (Fe 415) $\rightarrow$ $x_{u,max} = 0.48d.$

$$\boxed{M_{u,limit} = 0.138\ f_{ck}\ bd^2}$$

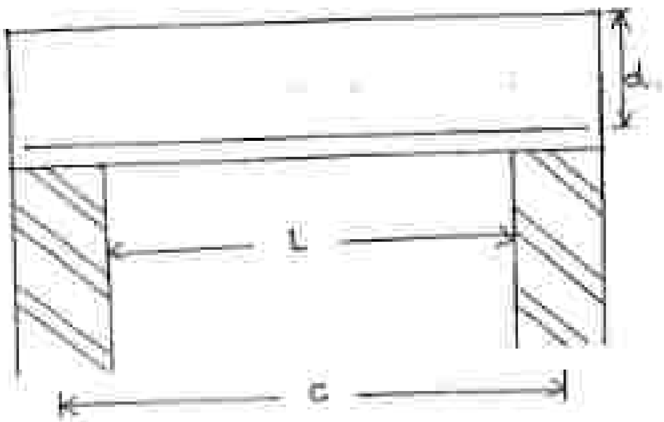
Eg: HYSD (Fe 500) $\longrightarrow$ $x_{u\,max} = 0.46d.$

$$\boxed{M_{u,limit} = 0.133\ f_{ck}\ bd^2}$$

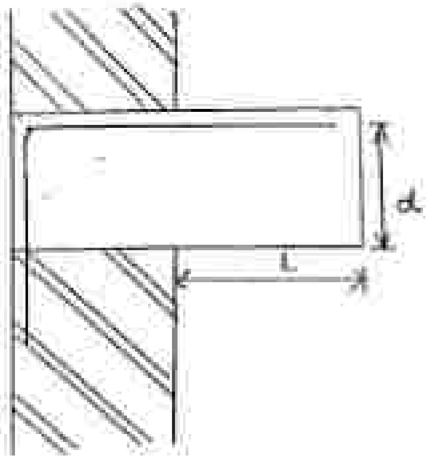
- The above simplified equations can be used only for balanced sections.
- In a beam it is better to use mild steel, but it is having bond problem being plane and round steel bars.

Among HYSD grades it is better to use Fe 415 grade. Fe 500 is better for columns.

→ Effective Span. (l)



$$l = \left.\begin{matrix} c \\ L+d \end{matrix}\right\} \text{less}$$



$$l = L + \frac{d}{2}$$

→ Minimum Reinforcement.

If steel provided less than min., there will be abrupt on sudden failure.

$$\boxed{\frac{(A_{st})_{min}}{bd} = \frac{0.85}{f_y}}$$

$$P_{min} = \frac{100\ (A_{st})_{min}}{bd} = \frac{0.85}{f_y} \times 100$$

Eg. Fe 415

$$P_{min}\ \% = \frac{0.85}{415} \times 100 = \underline{0.205\ \%}$$

→ Maximum Reinforcement.

(i) Beams.

$$\boxed{(A_{st})_{max} = 4\%\ A_g}$$

$$P_{max} = 4\%$$

(ii) Columns

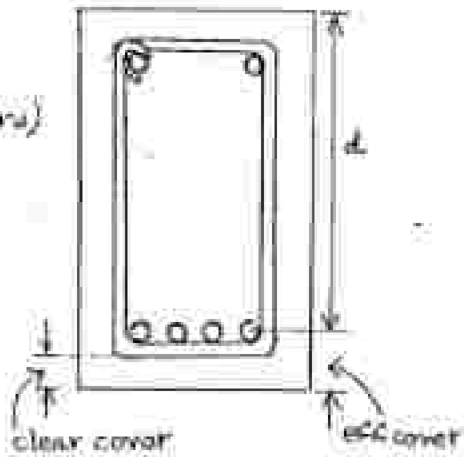
$$\boxed{(A_{st})_{max} = 6\%\ A_g}$$

$$P_{max} = 6\%$$

Max. steel is based on criticality in placing and compacting the concrete. With max percentage of steel, beam-column joint will be critical.

→ Cover (or) Clear Cover (or) Nominal Cover

- Clear cover is based on Serviceability (exposure or environmental conditions)



- Environmental conditions in India are categorised as below.

(i) Mild — 25 mm min cover

(ii) Moderate — 30 mm min cover.

(iii) Severe — 45 mm min cover (buildings in coastal areas, footings - touch with soil)

(iv) Very Severe — 50 mm min cover (footings in critical salty soils, buildings on shore)

(v) Extreme — 75 mm (off shore structures).

- Based on Structure :-

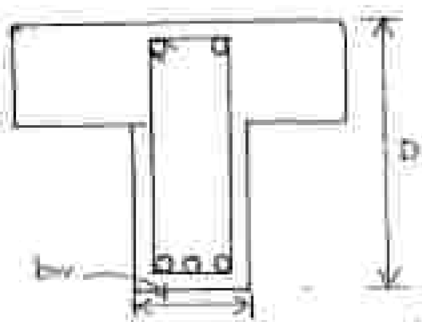
(i) Slabs — 20 mm min cover

(ii) Beams — 25 mm min cover

(iii) Columns — 40 mm min cover

(iv) Footings — 50 mm min cover.

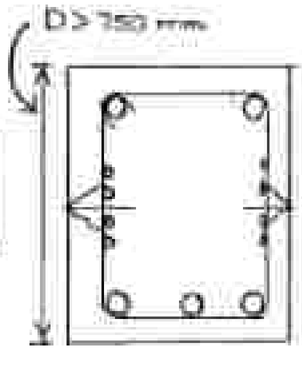
* side face rft (or) skin reinforcement.



$A_g = b_w D$

$$\boxed{(A_{st})_{SF} = 0.1\% \; A_g \text{ or } 0.1\% \; A_w}$$

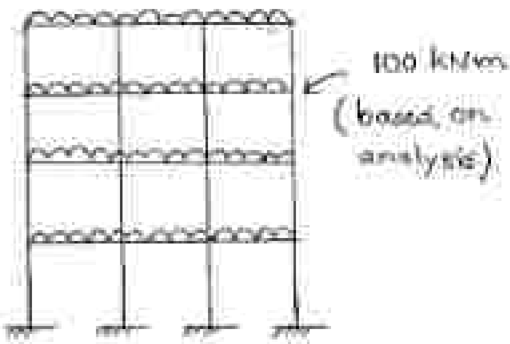
Spacing $\not>$ 300 mm c/c



In deep beams, lateral buckling occurs due to load, which causes torsion in the c/s. Maximum torsional shear stress develops on centre of side face. For resistance side face rft is provided.

→ Moment Redistribution:

Bending moment flows from higher magnitude point to lower magnitude point. As per IS 456, 30% of redistribution is allowed.



After 30% redistribution,

Design BM = 0.7 × 100 kNm

= 70 kNm

— Redistribution is not allowed in determinate members like simply supported bridge girders.

P-16

06. $b = 200$, $d = 400$, ·105 (#4, 20 ϕ), M20 Fe15

$x_{umax} = 0.53d = 0.53 \times d = 212$ mm.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 250 \times \frac{\pi}{4} \times 20^2 \times 4}{0.36 \times \cancel{20}^{15} \times 200}$$

$= \cancel{189.8}$ 253.073 mm

$x_u > x_{umax} \Rightarrow$ Over Reinforced section.

Load carrying capacity (or) moment of resistance of ORS should be reduced to that of balanced section using eqn ① & x_{umax}

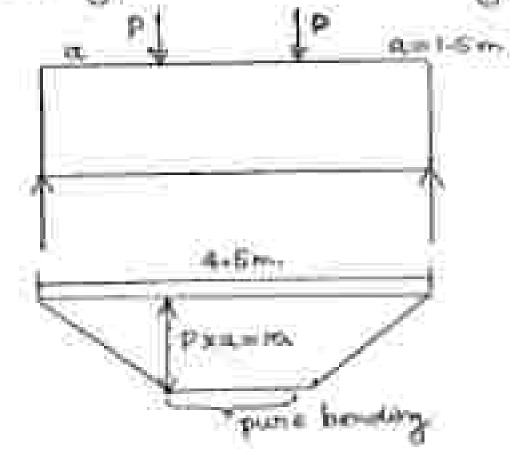
$M_{u,limit} = 0.148 f_{ck} bd^2 = 0.148 \times 15 \times 200 \times 400^2 = 71.2$ kNm

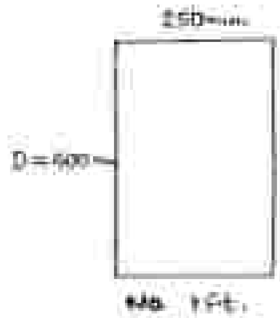
07 $$x_u = \frac{0.87\times 250\times \frac{\pi}{4}\times 20^2\times 3}{0.36\times 15\times 200} = 189.8 < x_{umax}$$

$$M_u = 0.36\, f_{ck}\, b\, x_u\,(d - 0.42\, x_u)$$

$$= 0.36\times 15\times 200\times 189.8\,(400 - 0.42\times 189.8) = \underline{65.6\ \text{kNm}}$$

08 Without reinforcement, the beam can be treated like homogeneous and bending equation can be used.

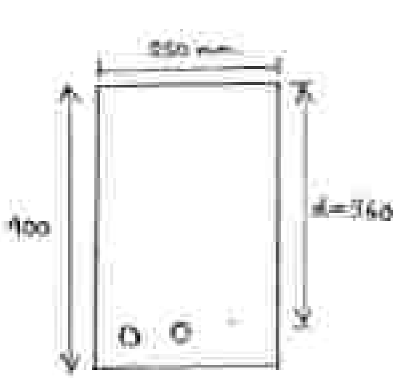




$$f = \frac{M}{Z} = \frac{P\times 1.5\times 1000}{\frac{250\times 400^2}{6}} \qquad \left\{\frac{E}{R} = \frac{M}{I} = \frac{f}{y}\right\}$$

$$\Rightarrow 2 = \frac{P\times 1.5\times 1000}{250\times\frac{400^2}{6}}$$

$$\therefore P = \underline{8.9}\ \text{kN}$$



09 $$x_{umax} = 0.48\,d = 0.48\times 360$$
$$= 172.8\ \text{mm}$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b} = \frac{0.87\times 415\times \frac{\pi}{4}\times 16^2\times 2}{0.36\times 20\times 250} = 80.66\ \text{mm}$$

$M_u = 0.36\, f_{ck}\, b\, x_{umax} (d - 0.42 x_u)$.

$= 0.36 \times 20 \times 250 \times 80.66 (340 - 0.42 \times 80.66)$

$\Rightarrow \underline{\underline{47.35}}$ kNm

$P \times 1.5 = 47.35$

$\Rightarrow P = \underline{\underline{31.56}}$ kN

Q1 $b = 200$, $d = 500$, M15, Fe415

$M_{u,lim} = 0.138\, f_{ck}\, bd^2$

$= 0.138 \times 15 \times 200 \times 500^2$

$= \underline{\underline{103.5}}$ kN

Q2 $C = T$ (balanced section).

$0.36\, f_{ck}\, b\, x_{umax} = 0.87\, f_y\, A_{st}$

$$A_{st} = \frac{0.36 \times 15 \times 200 \times (0.48 \times 500)}{0.87 \times 415} = \underline{\underline{420.74}}\ mm^2$$

Q3 $M = 0.138\, f_{ck}\, bd^2$ (Fe 415)

$138 \times 10^6 = 0.138 \times 20 \times 200 \times d^2$

$\Rightarrow d = \underline{\underline{500}}$ mm

Q4 $M = 0.133\, f_{ck}\, bd^2$ (Fe 500)

$149 \times 10^6 = 0.133 \times 20 \times b \times 500^2$

$\Rightarrow b = \underline{\underline{224}}$ mm

Q5 M20 & Fe 500, $b = 250$, $d = 500$, #4.16φ

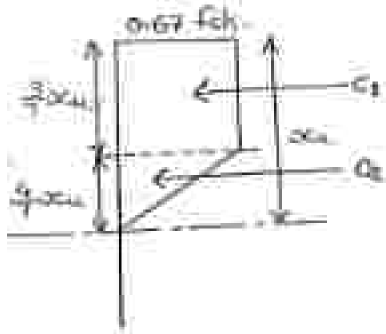
$\frac{x_{umax}}{d} = 0.46 \Rightarrow x_{umax} = 0.46 \times 500 = \underline{\underline{230}}$ mm

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b} = \frac{0.87\times 500\times 4\times\frac{\pi}{4}\times 16^2}{0.36\times 20\times 250}$$

$$= 194.36 \text{ mm}$$

$\Rightarrow x_u < x_{u,max}$

$\therefore$ Under reinforced section.



10. $C = C_1 + C_2$

$= 0.67 f_{ck}\times\frac{3}{7}x_u + \frac{1}{2}\times 0.67 f_{ck}\times\frac{4}{7}x_u$

$= 0.478\, f_{ck}\, x_u \times b$

$C = T$

$b\times 0.478 f_{ck}\, x_u = 0.87 f_y A_{st}$

$250\times 0.478\times 20\times x_u = 0.87\times 415\times 3\times\frac{\pi}{4}\times 20^2$

$\Rightarrow$ Depth of neutral axis, $x_u = \underline{142.207}$ mm

11. As per IS 456-2000,

Depth of neutral axis, $x_u = \dfrac{0.87 f_y A_{st}}{0.36 f_{ck}\, b}$

$$= \frac{0.87\times 415\times\frac{3}{4}\pi\times 20^2}{0.36\times 20\times 250}$$

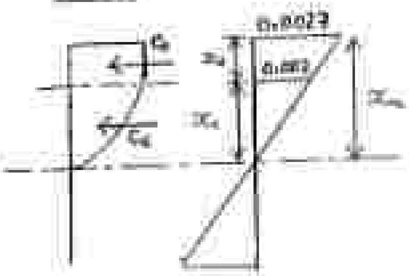
$= \underline{189.045}$ mm

Difference in neutral axis depths $= 189.045 - 142.207$

$= \underline{46.84}$ mm

12. $\dfrac{0.0027}{x_u} = \dfrac{0.002}{x_1}$

$x_1 = \dfrac{20}{27}x_u$ & $x_2 = \dfrac{7}{27}x_u$



$C = C_1 + C_2$

$= \left(\frac{7}{27} x_u \times 0.45 f_{ck} + \frac{2}{3} \times 0.45 f_{ck} \times \frac{20}{27} x_u\right) b$

$= 0.3389 f_{ck} x_u \times b$

$C = T$

$0.3389 \times 30 \times 300 \times x_u = 0.87 \times 250 \times \frac{\pi}{4} \times 2000$

$\Rightarrow x_u = \underline{142.623} \text{ mm}$

13. Force acting on compression zone $= C$

$= 0.3389 \times 30 \times 142.623 \times 300$

$= \underline{434.89} \text{ kN}$

14. $b = 150$ mm, $d = 350$ mm, $f_{ck} = 20$ MPa, $f_y = 415$ MPa

$M_{u,limit} = 0.138 f_{ck} b d^2$

$= 0.138 \times 20 \times 150 \times 350^2$

$= \underline{50.715} \text{ kNm}$

15. $M_{u,limit} = 0.87 f_y A_{st} (d - \overset{0.42}{} x_{umax})$

$50.715 \times 10^6 = 0.87 \times 415 \times A_{st} (350 - \overset{0.42}{} 0.48 \times 350)$

$\Rightarrow A_{st} = \underline{502.667} \text{ mm}^2$

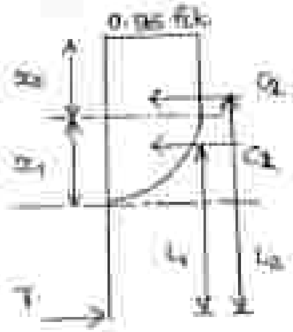
16. $\frac{0.003}{x_{umax}} = \frac{0.002 + \frac{f_y}{1.15 \times E_s}}{d - x_{umax}} \Rightarrow \frac{d - x_{umax}}{x_{umax}} = \frac{3.886 \times 10^{-3}}{0.003}$

$\frac{x_{u,max}}{d} = 0.4356$

Critical neutral axis depth, $x_{u,max} = 0.4356 \times 450$

$= \underline{126.04}$ mm

17. Design stress in concrete $= \dfrac{0.67 f_{ck}}{1.3} = 0.516 f_{ck}$



Design stress in steel $= \dfrac{f_y}{\gamma_m} = \dfrac{f_y}{1.1}$

$= 0.91 f_y$

From similar Δ^s of strain,

$$\frac{0.003}{x_u} = \frac{0.002}{x_1}$$

$$x_1 = \frac{2}{3} x_u \quad \& \quad x_2 = \frac{1}{3} x_u$$

$C_1 = (0.516 f_{ck} + 0) \frac{2}{3} \times b \times \frac{2}{3} x_u = 0.23 f_{ck}\, b x_u$

$C_2 = 0.516 f_{ck} \times b \times \frac{1}{3} x_u = 0.172 f_{ck}\, b x_u$

$T = \cancel{0.87}\ 0.909 f_y \times 3 \times \frac{\pi}{4} \times 16^2$

$= 548.55 f_y$

$$C_1 + C_2 = T$$

$$0.23 \times 35 \times 300 \times x_u + 0.172 \times 35 \times 300\, x_u = 548.55 \times 415$$

$$x_u = \underline{54.5} \text{ mm}$$

$M_u = C_1 L_1 + C_2 L_2$

$= C_1 \left(d - x_2 - \frac{3}{8} x_1\right) + C_2 \left(d - \frac{x_2}{2}\right)$

$= 131394.5 \left(450 - \frac{1}{3} x_u - \frac{3}{8}\left(\frac{2}{3} x_u\right)\right) + 98925.5 \left(450 - \frac{x_u}{6}\right)$

$= \underline{97.596}$ kNm

3rd Nov, Monday

04. DOUBLY REINFORCED SECTION

→ Need

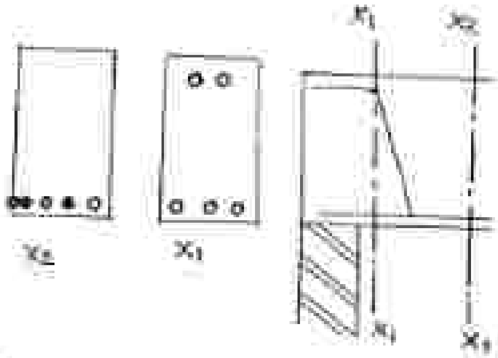
(i) Depth is restricted.

(ii) Reversal of stresses — Wind Load / Earth quake Load

(iii) Impact Load.

- due to moving wheel loads.
- blasting load
- wave effect of water.

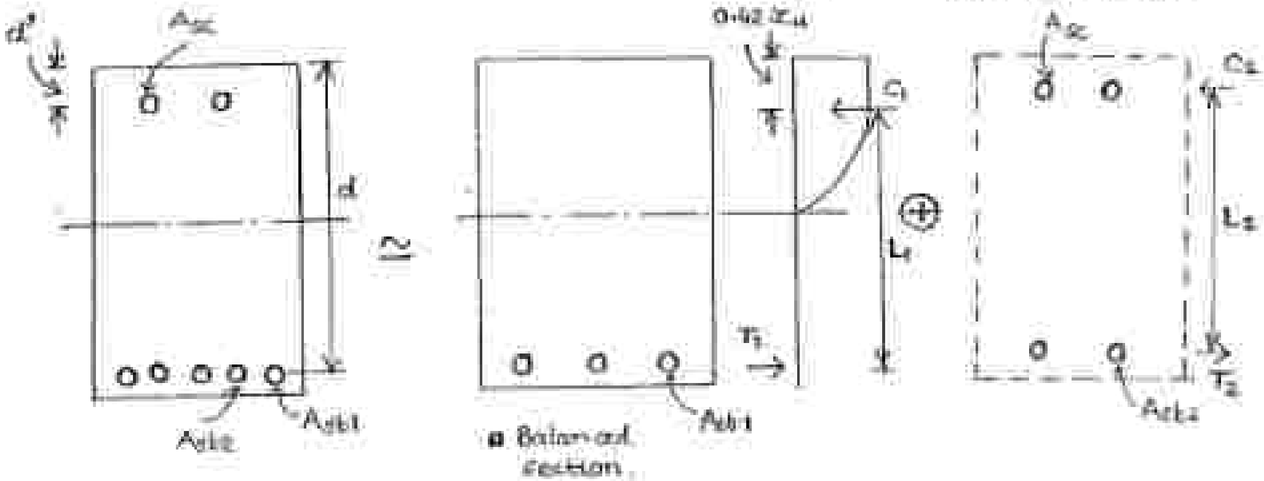
(iv) Support section of a beam.



(v) $M_u > M_{u,limit}$

M_u = BM due to external loads.

$M_{u,limit}$ = MR of a balanced singly reinforced section.



a Balanced section.

$$M_u = M_{u,lim} + (M_u - M_{u,limit})$$

→ Design of DRS

- Given Data includes:
  - (i) M_u
  - (ii) b, D, cover, d'
  - (iii) f_{ck}, f_y
- Required to calculate:
  - (i) A_{st} $(= A_{st1} + A_{st2})$
  - (ii) A_{sc}

(i) MR of a balanced section.

$$M_{u,limit} = (\dots) f_{ck} b d^2$$

(ii) If $M_u < M_{u,limit}$ ⇒ use SRS

(iii) If $M_u > M_{u,limit}$ ⇒ use DRS.

(iv) A_{st1} ⇒ steel required for a balanced section. (which is balancing concrete in compression zone)

$$M_{u,limit} = \text{eqn @ with } x_{umax}. \quad [illegible]$$

$$= 0.87 f_y A_{st1} (d - 0.42 x_{u,max}).$$

∴ A_{st1} is obtained

(v) A_{st2} ⇒ steel required for extra moment.

$$M_u - M_{u,limit} = T_2 L_2$$

$$= 0.87 f_y A_{st2} (d - d')$$

∴ A_{st2} is obtained

⇒ Total steel in tension, $A_{st} = A_{st1} + A_{st2}$

6(a)

(ii) $A_{sc} \Rightarrow$ steel required in compression for $M_u - M_{u,limit}$

$$M_u - M_{u,limit} = C_2 L_2$$

$$= \underbrace{f_{sc}}_{\text{steel compression}} (A_{sc})(d - d')$$

$f_{sc} \rightarrow$ stress in compression steel. ($\simeq 0.87 f_y$)

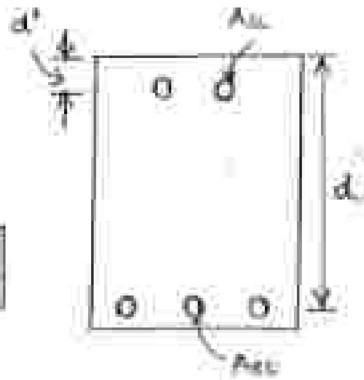
(Assuming compression steel is also yielded)

→ Analysis

(i) Given $\left.\begin{array}{l} b, D, \text{cover}, d' \\ A_{st}, A_{sc} \\ f_{ck}, f_y, f_{sc} \end{array}\right\}$ required to calculate M_R

Step 1: MR of balanced section ~~(singly reinforced)~~

$$x_{u\,max} = (\ldots) d.$$



Step 2: $x_u = ?$

$$C_1 + C_2 = T.$$

$$\boxed{0.36 f_{ck}\, b\, \underline{x_u} + f_{sc} A_{sc} = 0.87 f_y A_{st}}$$

Step 3: If $x_u < x_{u\,max} \Rightarrow$ under RS.

$MR \longrightarrow$ use x_u.

(i) Based on compressive forces

$$M_u = C_1 L_1 + C_2 L_2$$

$$\boxed{M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')} \rightarrow ③$$

(iii) Based on tensile forces.

$$M_u = T_1 L_1 + T_2 L_2$$

$$= 0.87 f_y A_{st1} (d - 0.42 x_u) +$$

$$0.87 f_y A_{st2} (d - d')$$

In this case the break up of main steel into A_{st1} & A_{st2} is not known. ∴ MR equation based on ~~tensi~~ compressive forces are used and not based on tensile force.

Step 4 : If $x_u = x_{u,max} \Rightarrow$ balanced section.

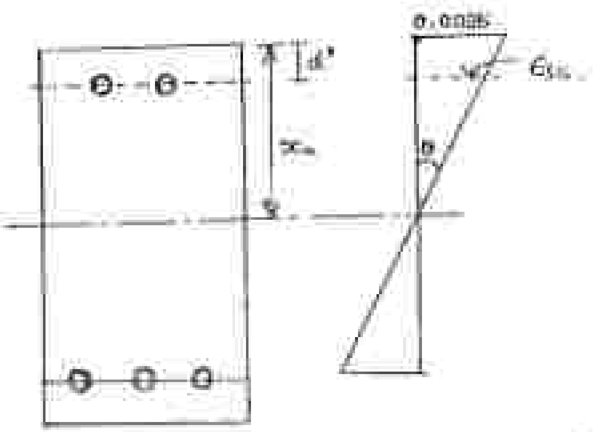
$$MR = M_{u,limit} = \text{equation } ③ \text{ with } x_{u,max}.$$

Step 5 : If $x_u > x_{u,max} \Rightarrow$ ORS.

Design of ORS is prohibited. If an existing beam found to be ORS, reduce its load or moment carrying capacity to that of a balanced section using $x_{u,max}$ in ③

P-20

01.



$$\frac{\epsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

$$\epsilon_{sc} = \left(1 - \frac{d'}{x_u}\right) 0.0035$$

02. For any grade of steel, max compressive stress $= 0.87 f_y$

93.
P-21

2 $b = 350$ mm, $d = 700$ mm, $d' = 50$ mm,

$f_{ck} = 15$ MPa, $f_y = 415$ MPa, $f_{sc} = 353.7$ MPa

$M_u = 300$ kNm $\times 1.5 = 450$ kNm.

$M_{u,limit} = 0.138 f_{ck} b d^2 = 0.138 \times 15 \times 350 \times 700^2$

$= 355$ kNm.

$M_u > M_{u,limit} \Rightarrow$ doubly reinforced section.

$M_{u,limit} = 0.87 f_y A_{st1} (d - 0.42 x_{u,max})$

$355 \times 10^6 = 0.87 \times 415 \times A_{st1} (d - 0.42 \times 0.48 d)$

$\Rightarrow A_{st1} = \underline{1759.3}$ mm^2

$M_u - M_{u,limit} = 0.87 f_y A_{st2} (d - d')$

$(450 - 355) 10^6 = 0.87 \times 415 \times A_{st2} (700 - 50)$

$\Rightarrow A_{st2} = 404.8$ mm^2

Tensile steel required, $A_{st} = A_{st1} + A_{st2}$

$= 1759.3 + 404.8$

$= \underline{2164.1}$ mm^2

$M_u - M_{u,limit} = f_{sc} A_{sc} (d - d')$

$95 \times 10^6 = 353.7 \times 10^6 \times A_{sc} (700 - 50)$

$A_{sc} = \underline{413.2}$ mm^2

Compression steel required $= \underline{413.2}$ mm^2

05. $b = 300$ mm ; $D = 500$ mm. $d' = 50$ mm, $f_y = 415$

$d = 500 - 37.5$
$= 462.5$ mm

$f_{sc} = 0.8566 f_y$
$= 355.489$

$f_c = 25$

$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5$ mm^2

$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.124$ mm^2

$x_{u\,max} = 0.48 \times 462.5$
$= 222$ mm.

$0.36 f_{ck}\, b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$.

$0.36 \times 25 \times 300\, x_u + 355.489 \times 402.124 = 0.87 \times 415 \times 1963.5$

$x_u = 209.62$ mm

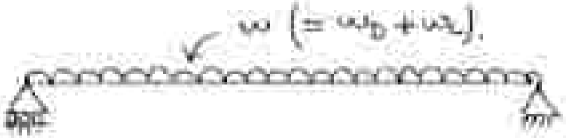
$\Rightarrow x_u < x_{u\,max} \rightarrow$ under reinforced section.

Ultimate moment of resistance, $M_u = 0.36 f_{ck}\, b x_u (d - 0.42 x_u)$
$+ f_{sc} A_{sc} (d - d')$.

$= 0.36 \times 25 \times 300 \times 209.62 (462.5 - 0.42 \times 209.62) + 355.489 \times 402.124 (462.5 - 50)$.

$= \underline{270.9}$ kNm

06. Working / service moment, $M = \frac{M_u}{\gamma_f} = \frac{270.9}{1.5}$
$= 180$ kNm.



Dead load, $w_d = \gamma_c \, bD$.

$= 25 \, kN/m^3 \times 0.3 \times 0.5$

$= 3.75 \, kN/m$.

$$\frac{(w_L + w_d) l^2}{8} = M_u$$

$$(w_L + 3.75) \times \frac{8^2}{8} = 180.6$$

$$\Rightarrow w_L = \underline{18.82} \; kN/m$$

$\therefore$ Superimposed live load $= \underline{\underline{18.82}}$ kN/m

03. $b = 300\,mm$, $d = 500\,mm$, $A_{st} = 2200\,mm^2$, $A_{sc} = 628\,mm^2$.
$d' = 50\,mm$.

$f_{ck} = 20$ MPa, $f_y = 250$, $f_{sc} = 0.87 f_y$ (tension & compression steel yield).

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + 0.87 \times 250 \times 628 = 0.87 \times \overset{250}{\cancel{415}} \times 2200$$

$$x_u = \underline{\underline{158.3}} \; mm$$

04. $M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$

$= 0.36 \times 20 \times 300 \times 158.3 (500 - 0.42 \times 158.3) + 0.87 \times 250 \times 628 (500 - 50)$

$= \underline{\underline{209.69}}$ kNm

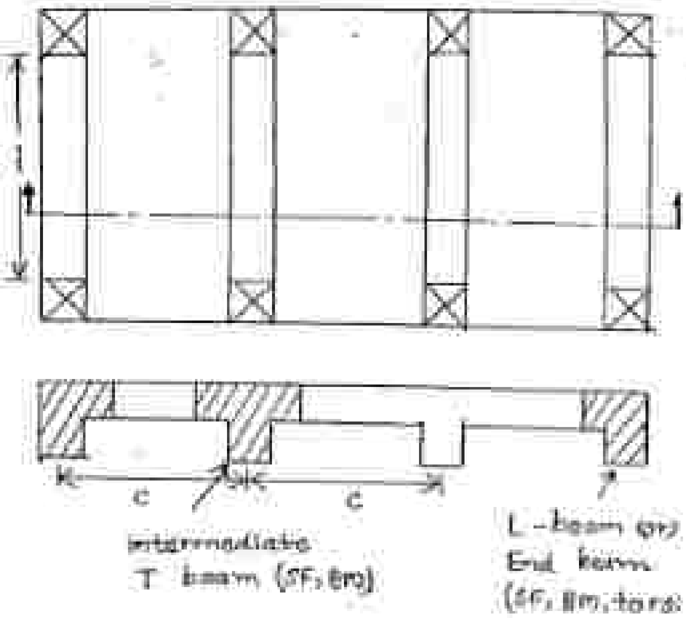
5th Nov,
Wednesday

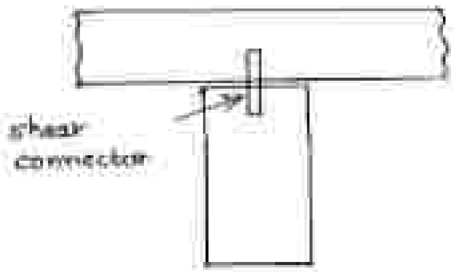
05. FLANGED BEAMS

→ Requirement

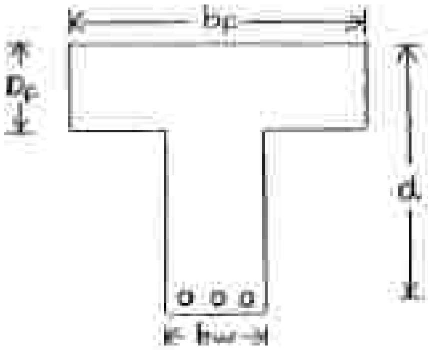
(i) monolithic construction (same joint)

(ii) Precast construction should have shear connectors.





• Pre-cast construction.



• c/s of SSB.

b_w → width of beam.

D_f → thickness of slab

b_f → effective width of flange

- Effective flange width can be calculated based on empirical formulae given in IS-456 so that the slab which is added to the beam should be in compression. (If the slab is in tension, its effect cannot be added

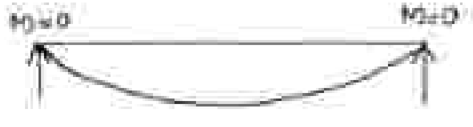
* Connected beams

(i) T-beam.

$$\boxed{b_f = \frac{l_o}{6} + b_w + 6D_f} \quad \not> c$$

$l_o \rightarrow$ c/c distance b/w zero BM points. (8)

19



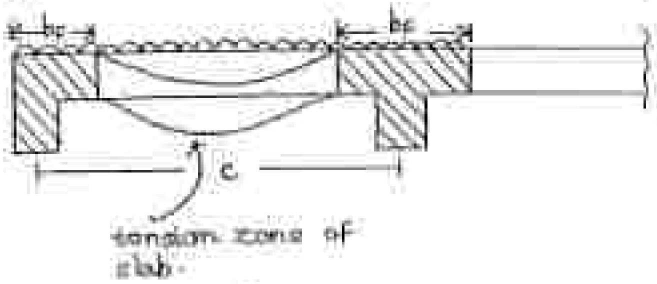
$\boxed{l_o = l}$

- SSB

$\boxed{l_o = 0.7l}$

- Fix / continuous beam.

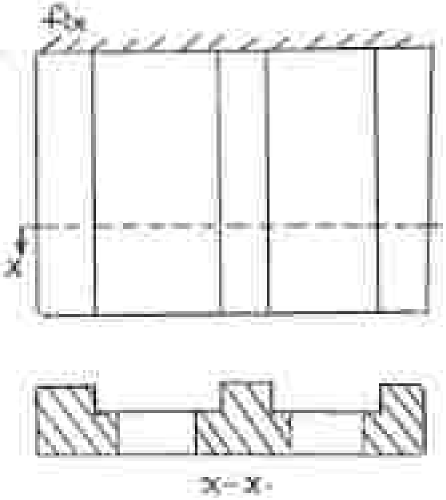
- Beyond l_o, slab is in tension. ∴ beam should not be designed like a flanged beam. Near the support, the beam is behaving like a doubly reinforced rectangular section with alternate bars bent up.
- Therefore, cantilever is also treated as ordinary rectangular beam only. ($l_o = 0$ for cantilever)
- If the slab on either side of the beam is in compression then only it can be added as a flange to the beam.



(ii) L - beam

$$b_f = \frac{l_o}{12} + b_w + 3D_f$$

$$\not> \frac{c}{2}$$



⊙ Inverted flanged beams used in cantilever portions.

Eg: Portico, porch.

→ Isolated beams

b → actual flange width available.

(i) T-beam.

$$\boxed{b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w}$$

$b_f \ngtr b$.

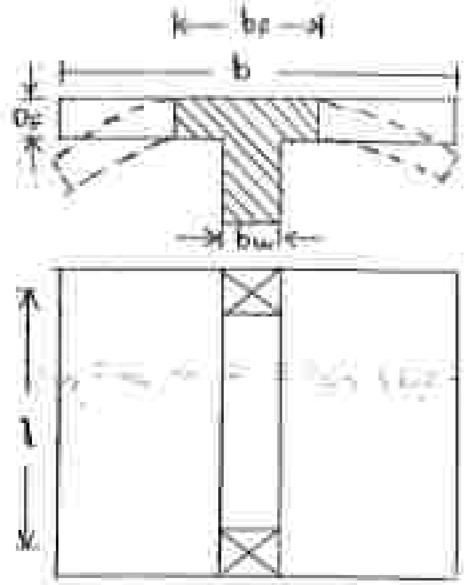
Eg: SS T-beam bridges

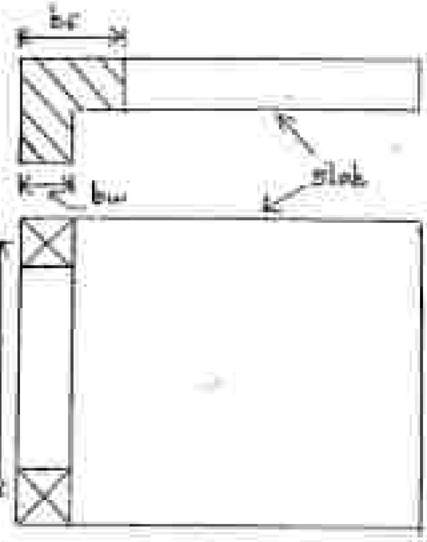
(ii) L-beam

$$\boxed{b_f = \frac{0.5\, l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w}$$

$b_f \ngtr b$

Eg: Portico, sunshade, chajjas





P-24

2. $b = 3m$, $l = l_0 = 6m$, $b_w = 0.25\,m$.

Foot bridge → Isolated T-beam. (SSB)

$$b_f = \frac{l_0}{\left(\frac{l_0}{b} + 4\right)} + b_w = \frac{6}{\left(\frac{6}{3} + 4\right)} + 0.25$$

$$= \underline{\underline{1.25 < b}}$$

P-25

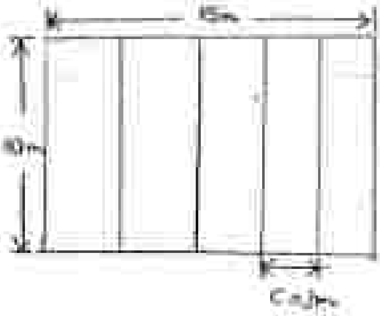
03 Beams are cast monolithic with columns (Fixed). 70

Intermediate beam ⇒ T-beam.

$b_w = 0.25$ m, $D_f = 0.1$ m, $b = 3$ m, $l_o = 0.7l$.
$= 0.7 \times 10 = 7$ m.

$$b_f = \frac{l_o}{6} + b_w + 6D_f$$

$$= \frac{7}{6} + 0.25 + 6 \times 0.1 = \underline{\underline{2.016\text{ m}}}$$



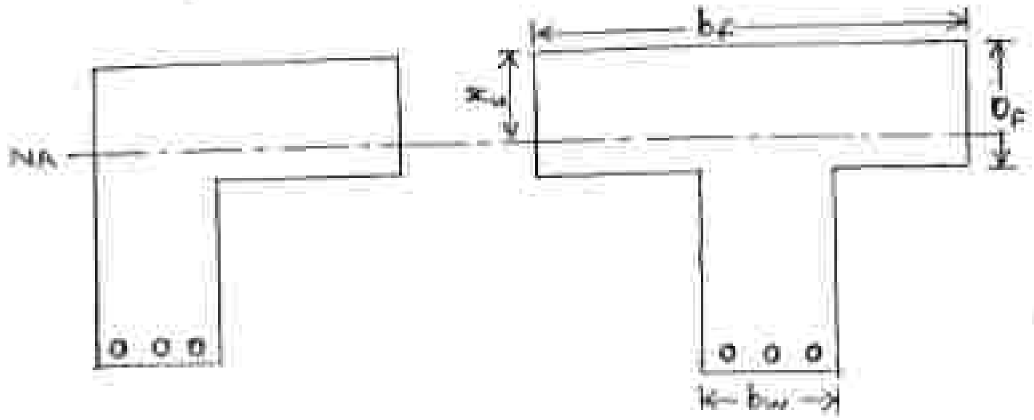
04. If beam is simply supported, $l_o = l$
$= 10$ m

End beam → L beam.

$$b_f = \frac{l_o}{12} + b_w + 3D_f$$

$$= \frac{10}{12} + 0.25 + 0.1 \times 3 = \underline{\underline{1.38\text{ m}}}$$

→ Analysis



(i) $x_{umax} = (..)d$

x_{umax} → critical / balanced / maximum / limiting NA.

MS → 0.53

Fe 415 → 0.48

Fe 500 → 0.46

(ii) Neutral Axis, x_u

Assume NA in flange $x_u \leq D_f$

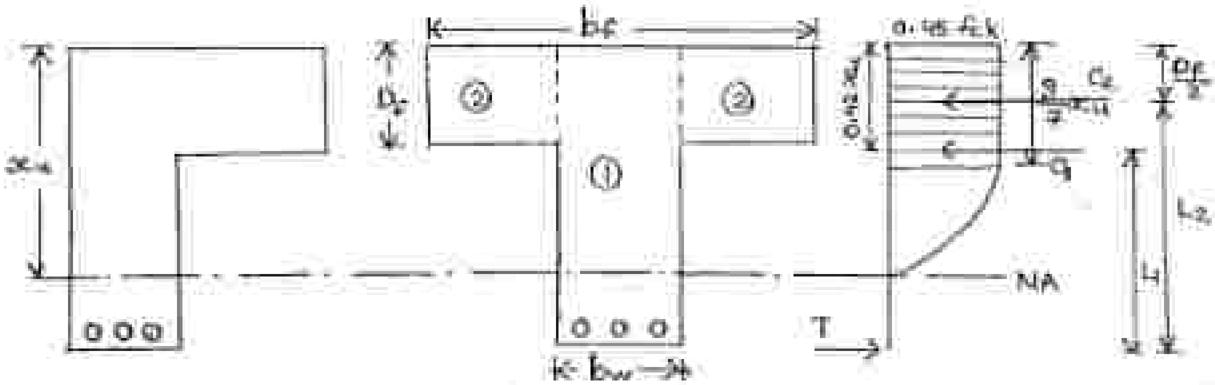
The effect of concrete should be considered only in compression zone. In this case, beam behaves like rectangle with $b = b_f$.

$$C = T$$

$$\boxed{0.36\, f_{ck}\, b_f\, x_u = 0.87\, f_y\, A_{st}}$$

If $x_u \leq D_f$, the assumption is correct. Further analysis is like a rectangular beam only with $b = b_f$.

If $x_u > D_f$, the assumption is wrong. Recalculate x_u by considering it in the web



(iii) NA in web.

Case 1: $D_f \not> \frac{3}{7} x_u$.

i.e. $$\boxed{\frac{D_f}{x_u} \leq \frac{3}{7} \text{ (or) } 0.43}$$

As per IS-456:

$$\frac{D_f}{d} \leq 0.2$$

In this case, the entire flange is with uniform stress of 0.45 fck, the web can be treated like a rectangular section. 7

$$C_1 + C_2 = T$$

$$\boxed{0.36\, f_{ck}\, b_w\, x_u + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}}$$

* Moment of Resistance, MR

– considering compressive forces,

$$M_u = C_1 L_1 + C_2 L_2$$

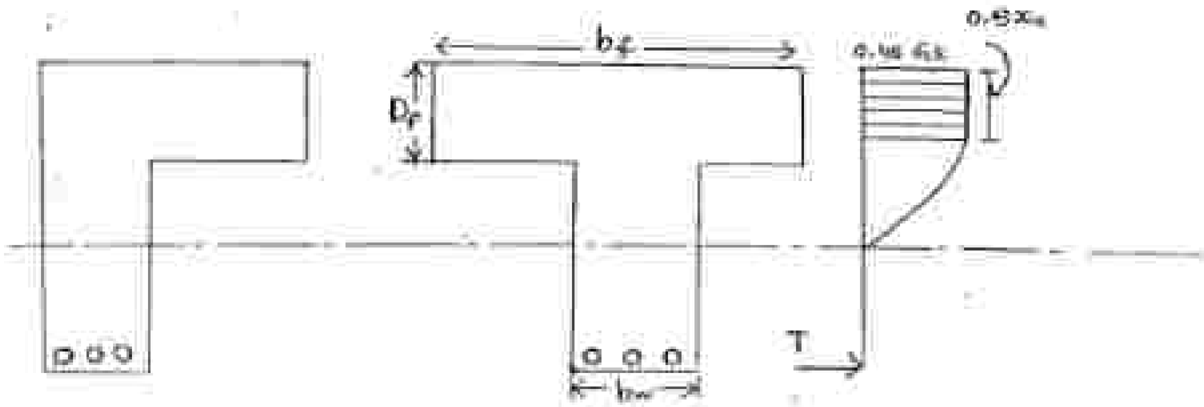
$$\boxed{M_u = 0.36 f_{ck}\, b_w\, x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)} \rightarrow (4)$$

MR equation based on tensile forces cannot be formulated directly as some portion of Ast is balancing concrete in web part and the other balancing concrete in flange part the break up is not known.

Case 2: $\frac{D_f}{x_u} > \frac{3}{7}$ (or) 0.43

As per IS code:

$$\boxed{\frac{D_f}{d} > 0.2}$$

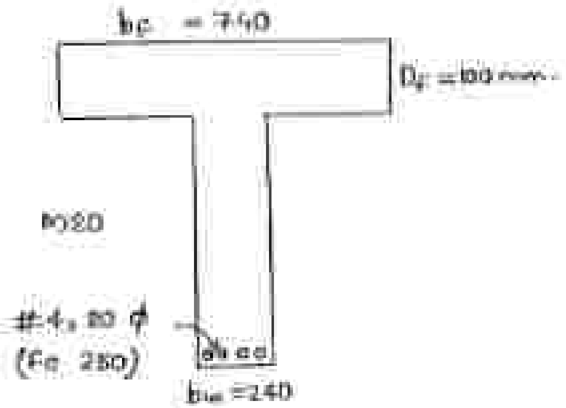


In this case, depth of flange is more than depth of rectangular portion of stress block. ∴ Some part of the flange is with uniform stress of 0.45 fck and the other less than 0.45 fck. In such a case, use the equations as in the above case, but replace an empirical constant y_f in place of D_f.

$$\boxed{y_f = 0.15\, x_u + 0.65\, D_f \;;\; \not> D_f}$$

P-25

01



$x_{umax} = 0.53\, d$

$= 0.53 \times 400$

$= 212$ mm.

$A_{st} = 1256.64$ mm^2

$\left\{ \dfrac{D_f}{d} = \dfrac{100}{400} = 0.25 \right\}$ If $x_u > D_f$

$0.36\, f_{ck}\, b_f\, x_u + \cancel{0.45} = 0.87\, f_y\, A_{st}$.

$0.36 \times 20 \times 740 \times x_u = 0.87 \times 250 \times 1256.64$.

$x_u = 51.3$ mm

$x_u < D_f \Rightarrow$ assumption is correct

$x_u < x_{umax} \Rightarrow$ under reinforced section

Analyse like a rectangular beam with $b = b_f$

$M_u =$ eqn 1 or 2 with x_u

$= 0.36\, f_{ck}\, b_f\, x_u (d - 0.42\, x_u)$

$= 0.36 \times 20 \times 740 \times 51.3 (400 - 0.42 \times 51.3)$

$= \underline{103.44}$ kNm

02. # 4 , 25 ϕ of Fe 415 . (28)

$0.36\, f_{ck}\, b_f\, x_u = 0.87\, f_y\, A_{st}$

$0.36 \times 20 \times 740 \times x_u = 0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2$

$\Rightarrow x_u = 133.06$ mm.

$x_u > D_f \Rightarrow$ assumption is wrong.

$\frac{D_f}{d} = 0.25 > 0.2$. (Case 2).

$y_f = 0.15\, x_u + 0.65 \times D_f$

$= 0.15\, x_u + 0.65 \times 100$

$0.36 \times f_{ck} \times b_w\, x_u + 0.45\, f_{ck}(b_f - b_w)\, y_f = 0.87\, f_y\, A_{st}$

$0.36 \times 20 \times \overset{240}{\cancel{740}}\, x_u + 0.45 \times 20(740 - 240)(0.15 x_u + 65) =$

$0.87 \times 415 \times 4 \times \frac{\pi}{4} \times 25^2$

$\overset{2403}{\cancel{6003}}\, x_u + 292500 = 708920$

$\Rightarrow x_u = \overset{173.3}{\cancel{69.368}}$ mm.

$x_{umax} = 0.48\, d = 0.48 \times 400$
$= 192$ mm

$y_f = 0.15 \times 173.3 + 65$
$= 90.99 < D_f$

$x_u < x_{umax} \Rightarrow$ under reinforced section.

$M_u = 0.36 \times 20 \times 240 \times 173.3 \times (\overset{400}{\cancel{740}} - 0.42 \times 173) + 0.45 \times 20 \times 500 \times 90.99 \left(\overset{400}{\cancel{740}} - \frac{90.99}{2}\right)$

$= \underline{\underline{243.14 \text{ kNm}}}$

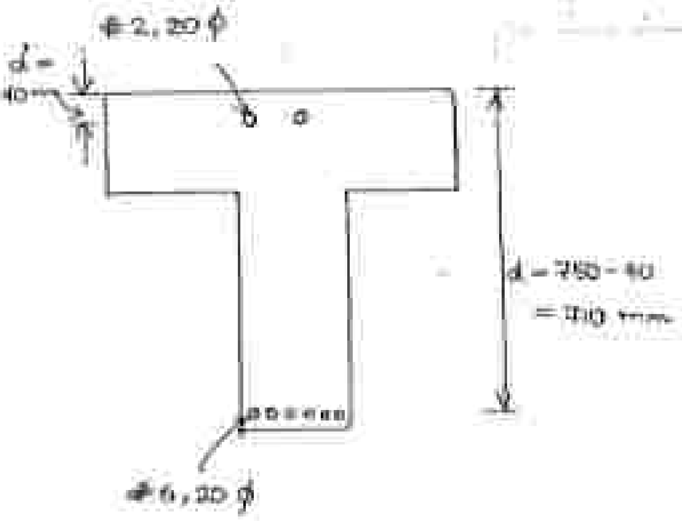
06. Effective flange width (SS Isolated T beam)

$b = 2m$ $\quad D_f = 150$ mm

$l = l_0 = 9m$ $\quad b_w = 300$ mm

$$b_f = \frac{l_0}{\left(\frac{l_0}{b}+4\right)} + b_w = \frac{9}{\left(\frac{9}{2}+4\right)} + 0.3$$

$$= \underline{\underline{1.36 \text{ mm}}} < b.$$



(i) $x_{umax} = 0.48\, d$

$= 0.48 \times 710$

$= \underline{\underline{340.8 \text{ mm}}}$

(ii) To find x_u.

Assume NA in flange,

$C_1 + C_2 = T$

$0.36 f_{ck} b_f x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}.$

$0.36 \times 20 \times 1360 \times x_u + 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 20^2 = 0.87 \times 415 \times 6 \times \frac{\pi}{4} \times 20^2$

$x_u = 37.06 \text{ mm} < D_f$

∴ Assumption is correct

$x_u < x_{umax} \Rightarrow$ URS.

07 Here $x_u < d'$ (cover). Compression steel will also enter into tension zone ∴ consider MR based on concrete in compression

not considered

$M_u = 0.36 f_{ck} b_f x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$

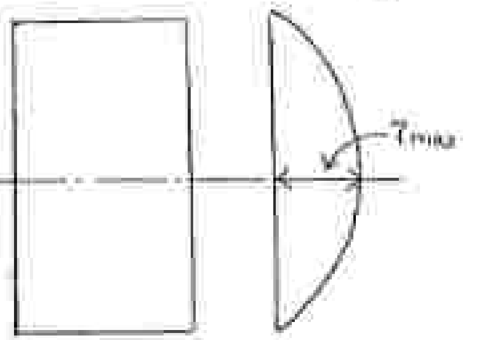
$= 0.36 \times 25 \times 1360 \times 37.06 (710 - 0.42 \times 37.06)$

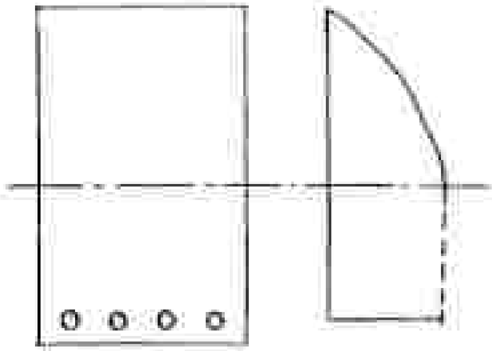
$= \underline{\underline{38 \text{ kNm}}}$

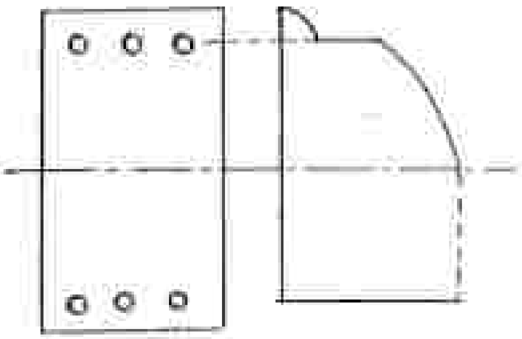
06 SHEAR

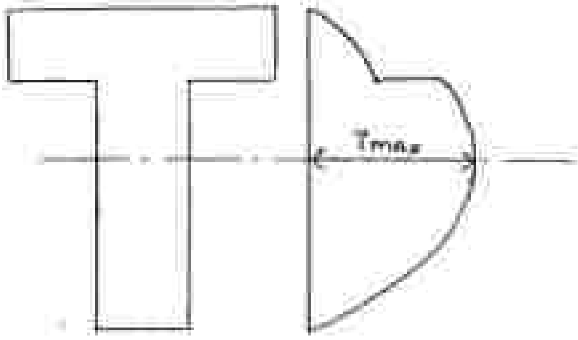
- Secondary design criteria
- Shear stress distribution in RCC beam.

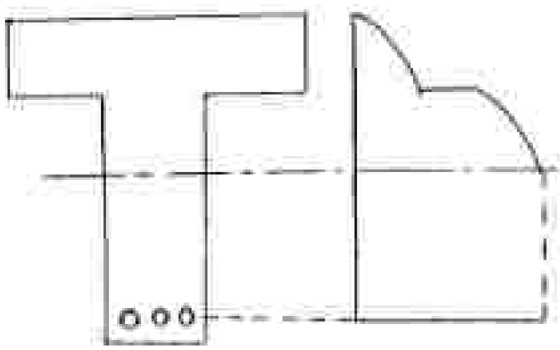
$$\tau = \frac{VA\bar{y}}{Ib} \; ; \; \downarrow b \Rightarrow \uparrow \tau$$

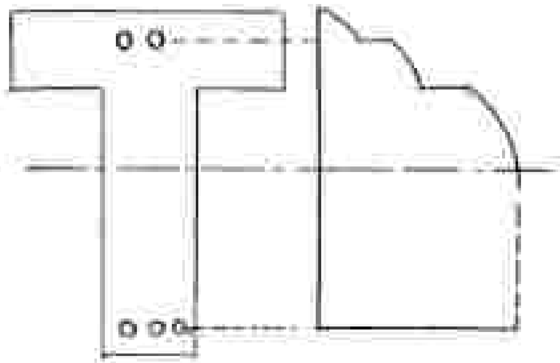


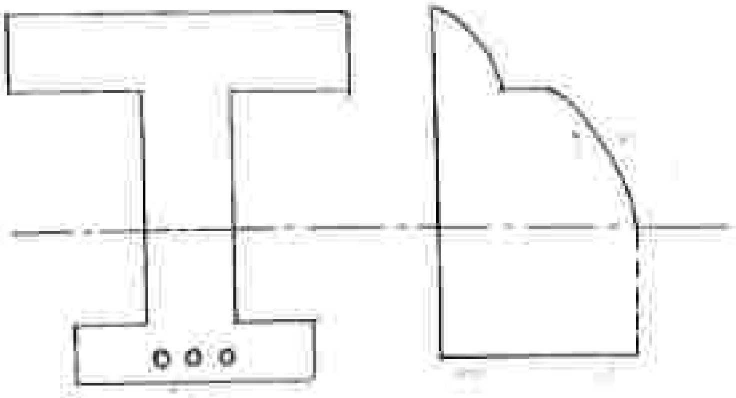






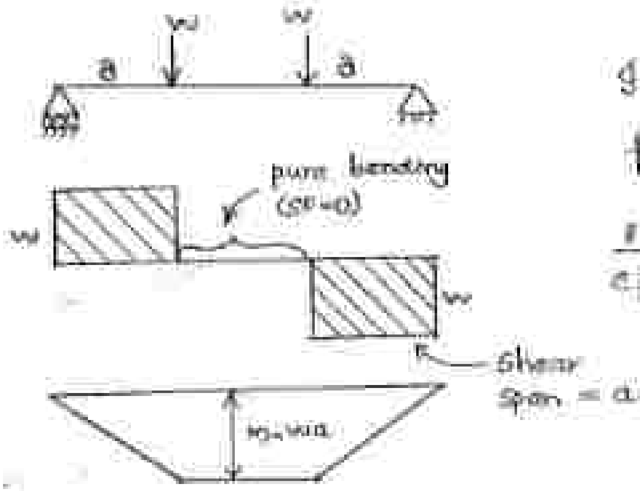






→ Shear Span.

Span in which shear force is a non zero constant value. In the shear span zone, bending moment need not be zero.



In lab, various types of beam failures can be assessed by,

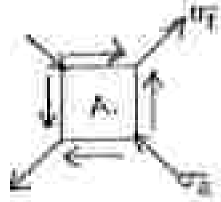
$\frac{\text{shear span}}{\text{effective depth}} \left(= \frac{a}{d}\right)$ ratio.

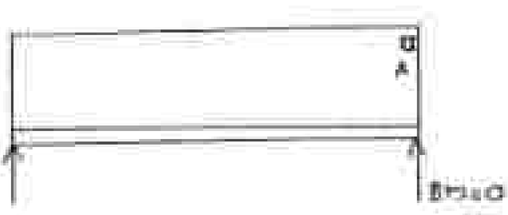
(1) Diagonal Compression Failure.

This failure occurs in the compression zone of a support.

$\sigma_1 = +\tau$ (diagonal tension)

$\sigma_2 = -\tau$ (diagonal compression)



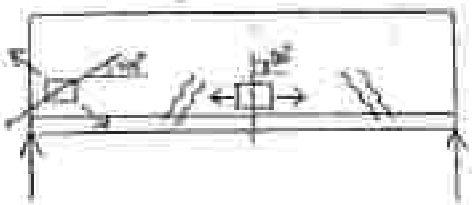


Concrete in compression zone get crushed due to heavy compressive force and then suddenly fails.

This failure can be rectified at the design stage itself with suitable measures

$$\boxed{\frac{a}{d} = 1 \text{ to } 2.5}$$ ⇒ Diagonal compression failure.

(ii) Diagonal tension failure



Diagonal tension crack can be prevented by providing proper shear rft. in beam

$$\boxed{\frac{a}{d} = 2.5 \text{ to } 6}$$ ⇒ Diagonal tension failure

(iii) Flexural Failure

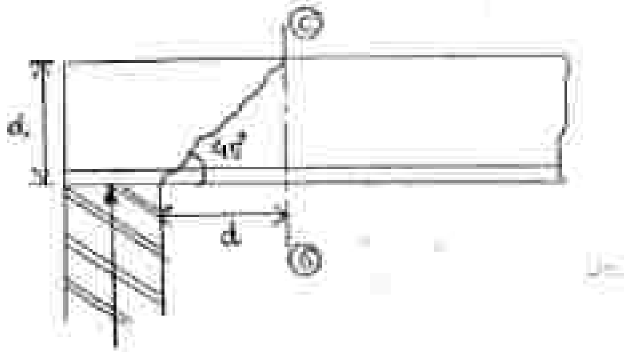
- occurs at central span
- 90° (vertical) crack develops
- It can be prevented by providing proper A_{st}

$$\boxed{\frac{a}{d} > 6}$$ ⇒ Flexural failure

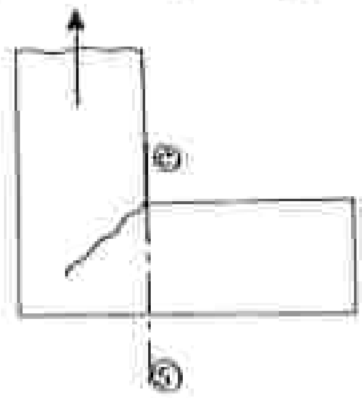
(iv) Flexural Shear Crack failure.

- rare crack
- 45–90° crack
- these are bending dominated shear cracks

→ Critical Section for Shear



If the support is under compression, critical section is at a distance of d from face of the supporting column.



If the support is under tension, then the critical section is exactly at the face of the support.

→ Shear Design

V_u
b, d (for flanged beams, $b = b_w$)
f_{ck}, f_y, τ_{cmax}, τ_c
} Given.

Step 1: Calculate nominal (avg) shear stress. (due to external loads)

$$\boxed{\tau_v = \frac{V_u}{bd}}$$

$\tau_v < \tau_{cmax}$ (no diagonal compression failure).

τ_{cmax} is maximum shear strength of concrete depends on grade of concrete given in IS 456.

If $\tau_v > \tau_{max} \Rightarrow$ diagonal compression failure occurs

$\therefore$ redesign by increasing d $\left(\tau_v \propto \frac{1}{bd}\right)$

- The most critical diagonal compression failure is prevented at the time of design itself by keeping $\tau_v \not> \tau_{c\,max}$.

$\tau_c \rightarrow$ shear resistance of RCC beam. (Inclusive of Ast)

τ_c depends on :

(i) Dowel action of Ast. (supporting action) $\rightarrow (\tau_{c1})$

(ii) Aggregate interlocking $\rightarrow (\tau_{c2})$

(iii) Uncracked concrete $\rightarrow (\tau_{c3})$

$$\tau_c = \tau_{c1} + \tau_{c2} + \tau_{c3}.$$

As per IS 456, τ_c can be calculated by P_t & f_{ck}

$$\boxed{P_t = \frac{100\,A_{st}}{bd}}$$

- If $\tau_v \leq 0.5\,\tau_c \Rightarrow$ safe in shear and no need of min. shear rft.

  (τ_v due to loads; τ_c resistance)

Eg: Slabs, lintels, chajjas, sunshade.

- If $\tau_v > 0.5\,\tau_c$ & $\tau_v < \tau_c \Rightarrow$ safe in shear, but to avoid future increase in shear due to secondary stresses due like temperature, shrinkage, creep etc. provide min. shear rft only in the form of vertical stirrups.

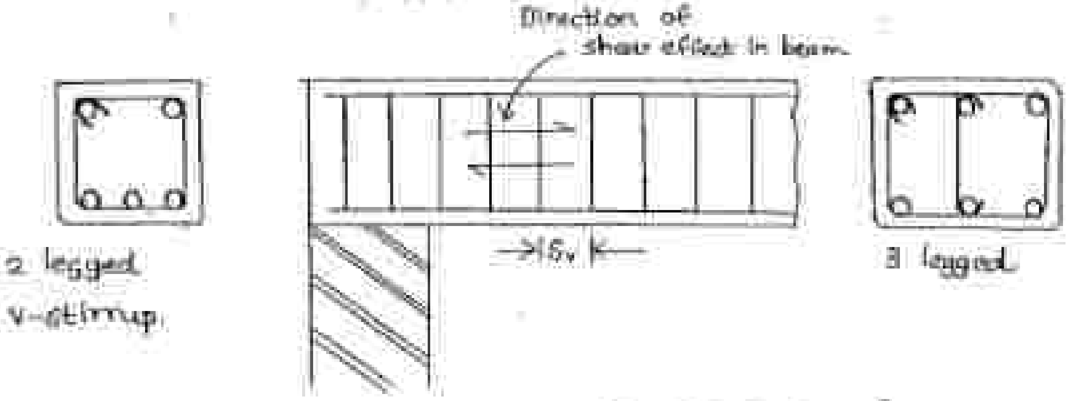
- Min shear reinforcement, $\frac{A_{sv}}{b\cdot S_v} = \frac{0.4}{0.87\,f_y}$

- Percentage min shear rft, $P_{vs} = \frac{100\,A_{sv}}{b\cdot S_v} = \frac{0.4\times100}{0.87\,f_y}$

Eg: 1) Fe 415 grade steel is used.

% min shear rft, $p_{ms} = \frac{0.4}{0.87 \times 415} \times 100$

$p_{ms} = 0.12\%$



$A_{sv} = n\left(\frac{\pi}{4}\phi^2\right)$

$S_v \leq 0.75\,d$
$S_v \leq 300$ mm. } use min.

where $n \rightarrow$ no. of legs.

$$\frac{A_{sv}}{b \cdot S_v} = \frac{0.4}{0.87\,f_y} \Rightarrow S_v = ?$$

⊙ If $\tau_v > \tau_c \Rightarrow$ not safe ; shear rft is required

1. Vertical stirrups
2. Inclined Stirrups.
3. Bent up bars.

– Most effective is inclined stirrup. But these are not stable in the formwork. ∴ in practice, vertical stirrups are commonly us

– Bent-up bars are optional. They cannot be used alone to resist shear, can be used with combination of v-stirrups or inclined stirrups

⊙ Inclined stirrups:

$\boxed{\alpha \geq 45°}$

01. $b = 230$ mm, $d = 400$ mm ; $f_y = 250$ MPa , $f_{ck} = 20$ MPa

$V_u = 120$ kN , $\tau_c = 0.48$ N/mm^2

2 legged stirrups of 8mm diameter.

$$\tau_v = \frac{V_u}{bd} = \frac{120\times10^3}{230\times400} = 1.304 > \tau_c.$$

$$V_{us} = V_u - \tau_c bd = 120\times10^3 - 0.48\times230\times400$$
$$= 75840$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87\times250\times2\times\frac{\pi}{4}\times8^2\times400}{75840} = \underline{115.32} \text{ mm}$$

$$S_v = \left.\begin{matrix} 0.75d = 300 \\ 300 \text{ mm} \\ 115.32 \end{matrix}\right\} \text{min} = \underline{115.32} \text{ mm}$$

02. Beam is subjected to Torque, $T_u = 10.9$ kNm

$$V_e = V_u + \frac{1.6 T_u}{b} = 120 + \frac{1.6\times10.9}{0.23} = \underline{195.83} \text{ kN}$$

$$V_{us} = V_e - \tau_c bd = 195.83 - 0.48\times10^{-3}\times230\times400 = \underline{\underline{151.667}} \text{ kN}$$

03. $b = 230$ mm, $d = 450$ mm, $\tau_{cmax} = 2.8$ MPa

$V_u = 50$ kN $\qquad \tau_c = 0.75$ MPa

$$\tau_v = \frac{V_u}{bd} = \frac{50\times10^3}{230\times450} = 0.48 < \tau_c \quad \text{but} > 0.5\,\tau_c.$$

$\therefore$ min shear reinforcement is provided

$$\frac{A_{sv}}{b\,S_v} = \frac{0.4}{0.87 f_y} \Rightarrow \frac{2\times\frac{\pi}{4}\times8^2}{230\times S_v} = \frac{0.4}{0.87\times250}$$

$$\therefore S_v = \underline{237.67} \text{ mm}$$

$$S_v = \min \text{ of } \begin{cases} 0.75d = 337.5\,mm \\ 300\ mm \\ 237.67 \end{cases} = \underline{\underline{237.67}}\ mm$$

04. $V_u = 100\ kN$

$$\tau_v = \frac{100 \times 10^3}{220 \times 450} = 0.966\ MPa > \tau_c$$

$$V_{us} = 100 - 0.75 \times 10^{-3} \times 220 \times 450$$
$$= 22.375\ kN$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$22375 = \frac{0.87 \times 250 \times 2 \times \frac{\pi}{4} \times 8^2 \times 450}{S_v}$$

$$\Rightarrow S_v = 439.75\ mm$$

$$S_v = \min \text{ of } \begin{cases} 439.75\ mm \\ 337.5\ mm \\ 300\ mm \\ 237.67\ mm \end{cases} = \underline{\underline{237.67}}\ mm$$

05. $V_u = 150\ kN$

$$\tau_v = \frac{150 \times 10^3}{220 \times 450} = 1.45 > \tau_c$$

$$V_{us1} = 0.87 f_y A_{sv} \times \sin\alpha$$
$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45° = \underline{\underline{102.66}}\ kN$$

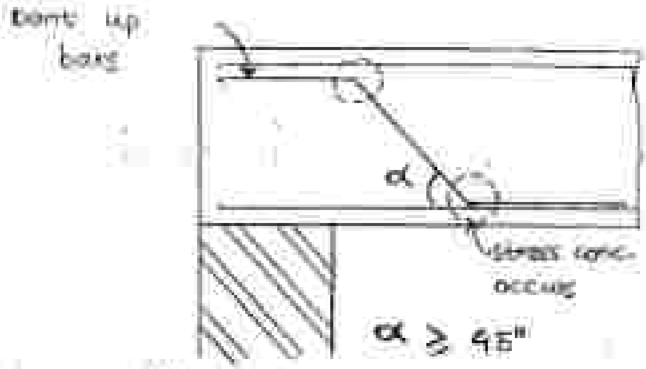
$V_{us1} \ngtr 0.5\ V_{us}$

$$V_{us} = V_u - \tau_c b d = 150 - 0.75 \times 10^{-3} \times 220 \times 450$$
$$= \underline{\underline{72.375}}\ kN$$

$$\therefore V_{us1} = 0.5\ V_{us} = \underline{\underline{36.1875}}\ kN$$

$$V_{us2} = \frac{0.87 f_y A_{sv} d}{S_v} \Rightarrow S_v = \underline{\underline{271.902}}\ mm$$

$$S_v = \min \text{ of } \begin{cases} 271.902\ mm \\ 337.5\ mm \\ 300\ mm \\ 237.67\ mm \end{cases}$$



→ Design SF for Shear Reinforcement

$$V_{us} = V_u - \tau_c bd$$

(a) For V-stirrups :

$$\boxed{V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}}$$

$S_v = ?$

(i) $S_v \not> (S_v)_{min}$ shear rft

(ii) $S_v \not> 0.75d$ (for V-stirrups) & $S \not> d$ (for inclined stirrups)

(iii) $S_v \not> 300$ mm.

} use minimum.

(b) For inclined stirrups

$$\boxed{V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin\alpha + \cos\alpha)}$$

(c) For Bent up bars

$$\boxed{V_{us1} = 0.87 f_y A_{sv} (\sin\alpha)}$$

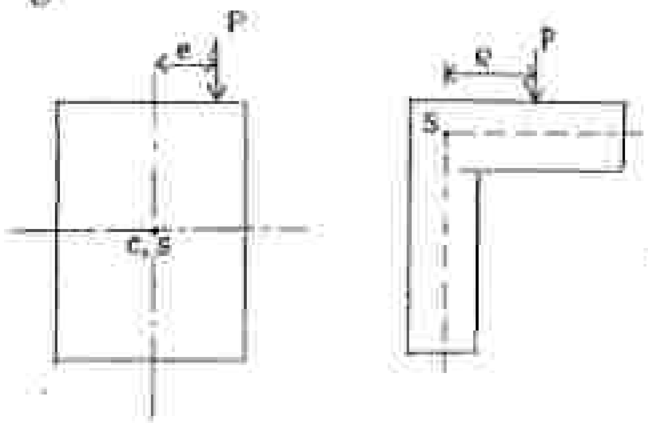
$V_{us1} \not> 0.5\, V_{us}$

Remaining $V_{us2} = V_{us} - V_{us1}$ (by stirrups).

6th nov,
THURSDAY

08. TORSION

If line of action of force is not passing through shear centre, then torsion develops in addition to shear force and bending moment.

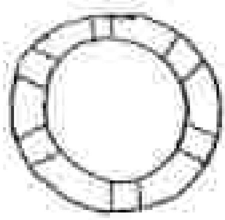


$T = Pe$

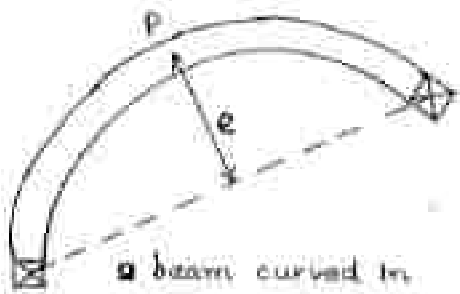
Torsion divided into:

1. Primary Torsion
2. Secondary Torsion

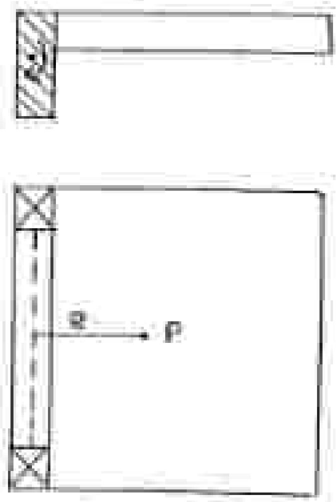
* Primary Torsion
  - design for torsion is compulsory.



▪ Ring beam of circular water tanks



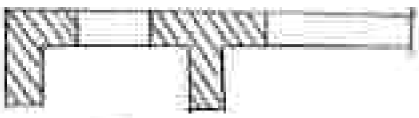
▪ beam curved in plan (curved in top view)



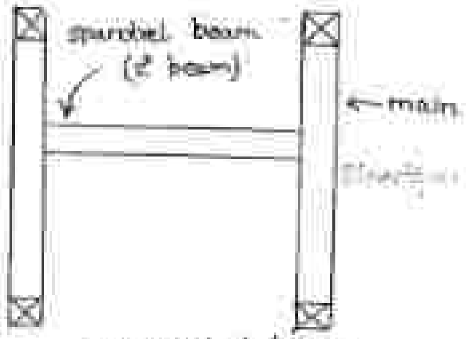
▪ Isolated L-beam

* Secondary Torsion

– Torsion design is optional.

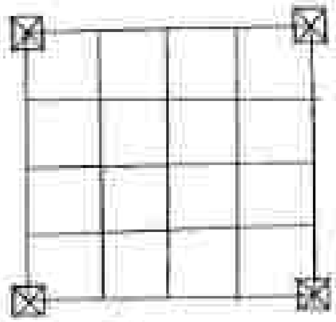


▪ connected beams



▪ spandrel beam

In the above case, bending of one member causes torsion in the other, but secondary torsion and the design is optional.



▪ grid floor

The torsion at the corner of two-way fixed (or) restrained slab is also secondary torsion

→ Torsion Design

— HSU based on Skew Bending theory.

— As per HSU, there is no separate design for torsion, some part of torsion is added to shear force and the other part is added to bending moment. With increased shear and bending, the member is designed. However, shear is primary and bending is secondary

– Given data includes :

⊙ T_u, M_u, V_u

⊙ b, D, cover to reinforcement

⊙ f_{ck}, f_y, τ_{cmax}, τ_c

Step 1: Equivalent shear force is calculated.

$$V_e = \underbrace{V_u}_{\text{due to external loads}} + \underbrace{V_T}_{\text{due to torsion}}$$

$$= V_u + \frac{1.6\,T_u}{b}$$

$$V_e = V_u + \frac{1.6\,T_u}{b}.$$

Step 2: Nominal (average) shear stress.

$$\tau_{ve} = \frac{V_e}{bd}$$

For L beam, $b = b_w$

ⓐ $\underbrace{\tau_{ve}}_{\text{due to loads}} \not> 0.5\,\underbrace{\tau_c}_{\text{resistance.}}$

Safe in shear; no need of even min shear reinforcement.

Eg: Sunshade, chajja.

ⓑ $\tau_{ve} > 0.5\,\tau_c$ but less than ($\leq$) τ_c

Safe in shear,
But min shear rft is required. by vertical stirrups only.

$$\frac{A_{sv}}{b \cdot S_v} = \frac{0.4}{0.87\,f_y}$$

$$S_v = ? \qquad \left.\begin{aligned} S_v &\not> 0.75\,d \\ &\not> 300\text{ mm} \end{aligned}\right\}\text{ use minimum}$$

ⓒ $\tau_{ve} > \tau_c$

Not safe in shear.
Shear rft. is required only in the form of vertical stirrups
(as per IS 456, inclined stirrups & bent up bars should not be consid

* Design SF for Shear Reinforcement

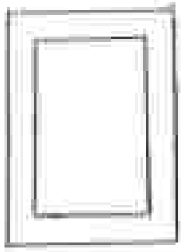
$$\boxed{V_{us} = V_u - \tau_c bd}$$

- For V stirrups,

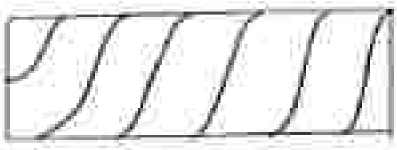
$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$; not applicable If torsion is acting !!

$$A_{sv} = \frac{S_v}{0.87 f_y d_1}\left(\frac{T_u}{b_1} + \frac{V_u}{2.5}\right)$$; not asked usually...

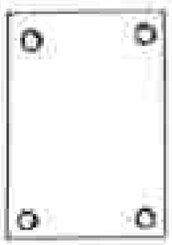
P-40

2. 

← Box girder (effective to resist torsion)

3. 

Brittle material cracks due to torsion → spiral @ 45°

4. 

One main steel bar @ each corner.
Total 4 bars required.

→ Main Reinforcement (A_{st}) in a beam with Torsion:

- If $\tau_{ve} \leq \tau_c$, then design BM = M_u

M_u → BM due to external loads.

- If $\tau_{ve} > \tau_c$, then design BM for A_{st}

$$M_{e1} = M_u + M_T$$

$$\boxed{M_{e1} = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}}$$

— overall depth.

- A_{sc} (compression steel) is also required if

$$M_T > M_u$$

$$\boxed{M_{e2} = M_T - M_u}$$

- If $M_T < M_u$, compression steel is not required

P-42

4. $b = 500$ mm, $D = 700$ mm, $d = 700 - 35$
$= 665$ mm

Factored shear force = ultimate shear force, V_u
$= 15$ kN.

$M_u = 100$ kNm, $T_u = 10$ kNm, $\tau_c = 1.5$ MPa $= 1.5 \times 10^{-3}$ kN/mm^2

$$V_e = V_u + \frac{1.6\,T_u}{b}$$

$$= 15 + \frac{1.6 \times 10}{0.5} = 47 \text{ kN.}$$

$$\tau_{ve} = \frac{V_e}{bd} = \frac{47}{0.5 \times 0.665} = 141.353 \text{ kN/m}^2 = 1.41 \times 10^{-4} \text{ kN/mm}$$

NOTE :

- If τ_c is given in the problem, then compare with τ_{ve} and decide design BM.
- If τ_c is not given, then directly consider, design BM as M_{e1} for main steel (A_{st}).

$$\tau_{ve} = 0.141 \text{ MPa} < \tau_c \, (= 1.5 \text{ MPa}).$$

$\therefore$ Design BM $= M_u = \underline{\underline{100 \text{ kNm}}}$

5. $M_u = 200$ kNm, $V_u = 20$ kN, $T_u = 9$ kNm

$b = 300$ mm, $D = 425$ mm, $d = 425 - 25 = 400$ mm.

$$V_e = V_u + \frac{1.6\, T_u}{b}$$

$$= 20 + \frac{1.6 \times 9}{0.3} = \underline{\underline{68 \text{ kN}}}$$

6. Given $\tau_{ve} < \tau_c$.

$$\therefore M_{e1} = M_u = \underline{\underline{200 \text{ kNm}}}$$

7. Critical section is at a distance, d (effective depth) from support.

Design SF = SF @ critical section

$$= \frac{10 \times 10}{2} - 10 \times d = 50 - 10 \times 1$$

$$= \underline{\underline{40 \text{ kN}}}$$

1. $b = 300$ mm, $D = 1000$ mm,

$V_u = 150$ kN, $M_u = 150$ kNm, $T_u = 30$ kNm.

$$V_e = V_u + \frac{1.6\, T_u}{b} = 150 + \frac{1.6 \times 30}{0.3} = \underline{\underline{310 \text{ kN}}}$$

Equivalent $BM = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7}$

$$= 150 + \frac{30\left(1 + \frac{0.1}{0.3}\right)}{1.7} = \underline{\underline{226.47}} \text{ kNm}$$

02. $b = 0.3m$, $D = 0.6$ m, $M_u = 100$ kNm, $T_u = 34$ kNm, $V_u = 100$ kN

$$M_{e1} = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7} = 100 + \frac{34\left(1 + \frac{0.6}{0.3}\right)}{1.7}$$

$$= \underline{\underline{160}} \text{ kNm}$$

3. $T_u = 68$ kNm

$$M_T = M_u + \frac{T_u\left(1 + \frac{D}{b}\right)}{1.7} = 100 + \frac{68\left(1 + \frac{0.6}{0.3}\right)}{1.7}$$

$$= 120 \text{ kNm.}$$

$M_T > M_u$

$\Rightarrow M_{e2} = M_T - M_u = 120 - 100 = \underline{\underline{20 \text{ kNm}}}$

10. COLUMNS
COMPRESSION

* Slenderness ratio, $\lambda = \dfrac{l}{r_{min}}$

For RCC,

$$\boxed{\text{Simplified } \lambda = \frac{l}{b}}$$

$l \rightarrow$ effective length

$b \rightarrow$ least lateral dimension

- Compression Members
  - Pedestal: $\frac{l}{b} \leq 3$
  - Columns
    - Short columns: $\frac{l}{b}$: 3 to 12; $3 < \frac{l}{b} \leq 12$
      1. Axially Loaded
      2. Axially Loaded + uni-axial BM.
      3. Axially Loaded + bi-axial BM.
    - Long columns: $12 < \frac{l}{b} \leq 60$

→ Minimum Eccentricity

$$\boxed{e_{min} = \frac{L}{500} + \frac{b}{30}} \text{ ; min of 20 mm}$$

$b \rightarrow$ least lateral dimension (c/s) ; $L \rightarrow$ unsupported length

→ Short - axially Loaded Columns $\left(3 < \frac{l}{b} \leq 12 \right)$:

Practically, applying the axial load is difficult. ∴ IS 456 given little consideration. As long as eccentricity of the column load is not exceeding, e_{min} given by the code, the column can be treated as axially loaded column only.

$$\boxed{\begin{aligned} P_u &= P_c + P_{sc} \\ &= 0.4 f_{ck} A_c + 0.67 f_y A_{sc} \end{aligned}}$$

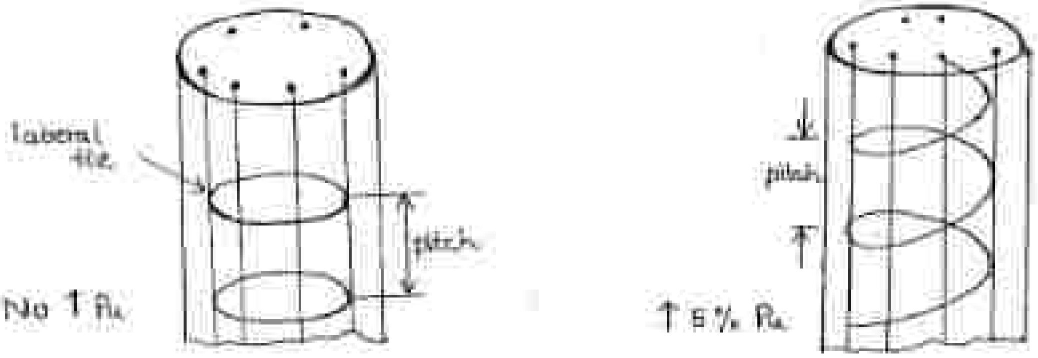
To avoid sudden crushing problem of column, the permissible stresses are reduced. $\left(\frac{f_{ck}}{2.5} \; \& \; \frac{f_y}{1.5} \right)$

L → unsupported length. It is the length over which column has no lateral support.

l → effective length, c/c distance b/w two successive zero BM points

→ Circular column with Helical Reinforcement

$$\boxed{\begin{aligned} P_u &= \uparrow 5\% \; P_u \\ &= 1.05 \left(0.40 f_{ck} A_c + 0.67 f_y A_{sc} \right) \end{aligned}}$$

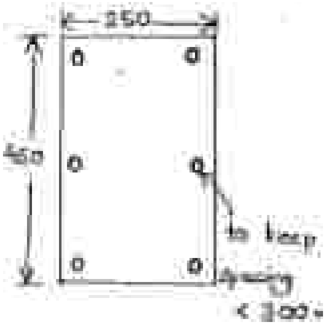


The purpose of lateral tie (or) helical rft is to support main steel or longitudinal rft

Helical rft is more effective because of its continuity

* Min % longitudinal reinforcement = 0.8%
(to prevent crushing of concrete)

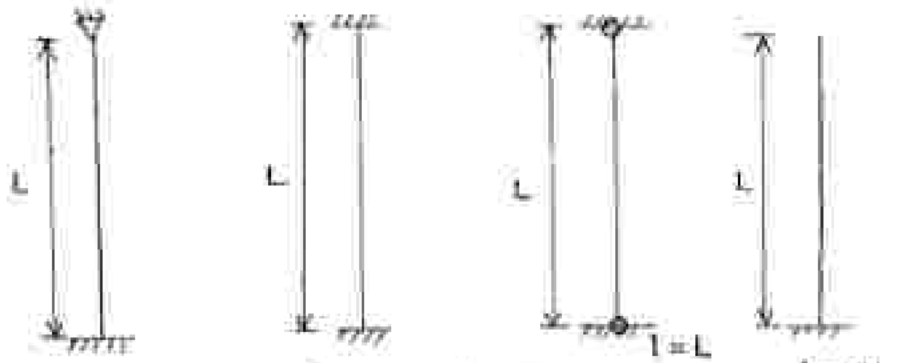
* Max reinforcement = 6%
(to avoid congestion)



* Min. size of longitudinal bar = 12 mm
(to avoid buckling of bars)

* Spacing of longitudinal bars ≤ 300 mm

→ Effective Length (l)



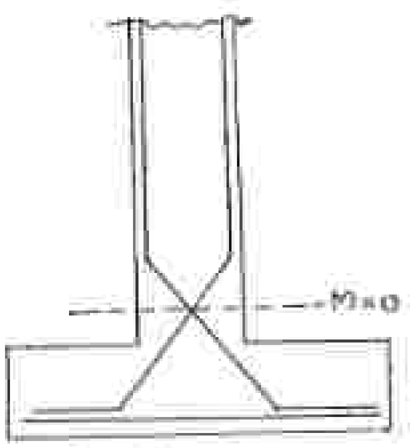
l = 0.8L (0.707L)

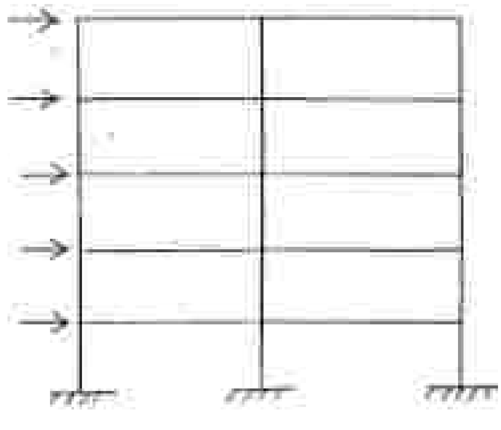
l = 0.65L (0.5L)

l = L

l = 2L

(Practical values are slightly increased than theoretical values)





▪ RCC Hinge.

NOTE:

- In case of a framed structure, subjected to critical sway loads, the effective lengths are based on WOOD'S TABLES given in IS 456.

- Effective length of a column is independent of load acting on the column.

P-54

a) $b \times d = 300\,mm \times 600\,mm$

$$P_u = P_c + P_{sc}$$

$$= 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

Min % steel, $(A_{sc})_{min} = 0.8\,\%\,A_g$

$$= \frac{0.8 \times 300 \times 600}{100} = 1440\,mm^2$$

$$A_c = A_g - A_{sc}$$

$$= 300 \times 600 - 1440 = 178560$$

$$P_u = 0.4 \times 20 \times 178560 + 0.67 \times 415 \times 1440$$

$$= \underline{\underline{1828.87}}\ kN$$

a2. $A_g = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 300^2 = 70685.83\ mm^2$

$A_{sc} = 1\%\ A_g = \frac{1}{100} \times 70685.83 = 706.86\ mm^2$

$A_c = A_g - A_{sc} = 69978.97\ mm^2$

$$P_u = 1.05\,(0.4 \times 20 \times A_c + 0.67 \times 415 \times A_{sc}).\ \left\{\begin{array}{l}\text{circular} \\ \text{column with} \\ \text{helical rft}\end{array}\right\}$$

$$= \underline{\underline{794.2}}\ kN$$

33

3. $b \times d = 300 \text{ mm} \times 300 \text{ mm}$

Given $A_c = bd$.

$A_{sc} = 4 \times \frac{\pi}{4} \times 20^2$

$\therefore P_u = 0.4 \times 20 \times 300^2 + 0.67 \times 415 \times 4 \times \frac{\pi}{4} \times 20^2$

$= \underline{\underline{1069.41}}$ kN

4. If support conditions are not given, use $l = L = 3$ m.
(hinge-hinge)

$\frac{l}{b} = \frac{3000}{450} = 6.67 < 12 \Rightarrow$ short

$\frac{l}{D} = \frac{3000}{600} = 5 < 12 \Rightarrow$ short.

5. Compatibility condition.

$dl_c = dl_s$

$\frac{P_c \times l_c}{A_c \times E_c} = \frac{P_s \times l_s}{A_s \times E_s}$

Given, modular ratio, $m = \frac{E_s}{E_c} = 10$

$A_{sc} = 1\% \ A_c$ (1% net c/s area)

For composite column, $l_s = l_c = l$.

$\frac{P_c \times l}{A_c \times E_c} = \frac{P_s \times l}{\frac{A_c}{100} \times 10 E_c}$

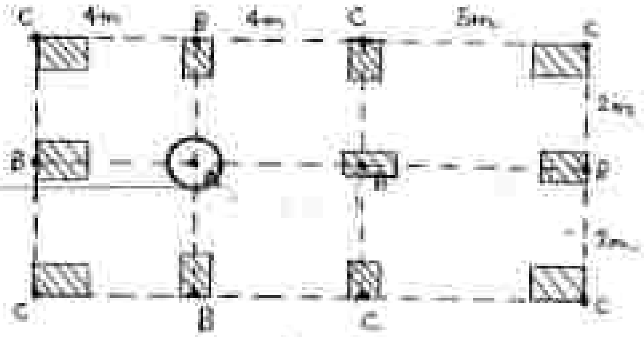
$\frac{P_s}{P_c} = \frac{10}{100} = 0.1 \times 100 = \underline{\underline{10\%}}$

→ Short Axially Loaded Column with Uni-axial BM

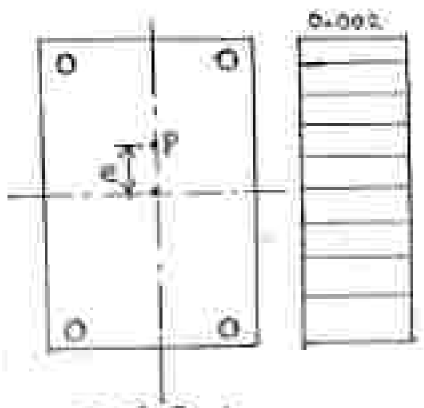
A → axially loaded.

B → axially loaded + uni-axial BM.

C → axially loaded + bi-axial BM.

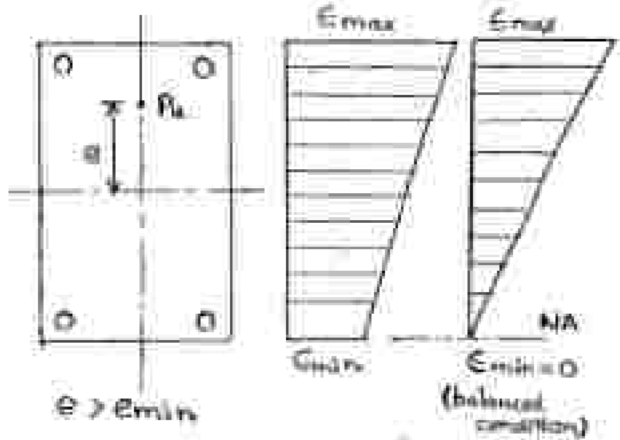


• If column of category C with equal moments on either side, should be oriented based on sway of the building frame



$e \leq e_{min}$.

(Axially Loaded Column)

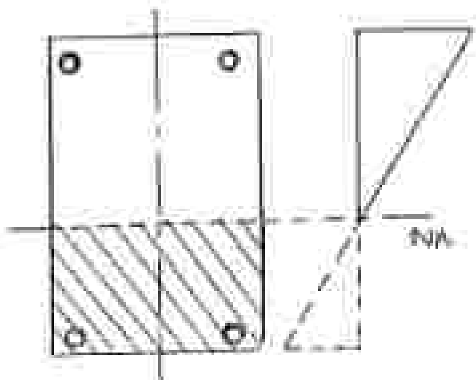


$e > e_{min}$

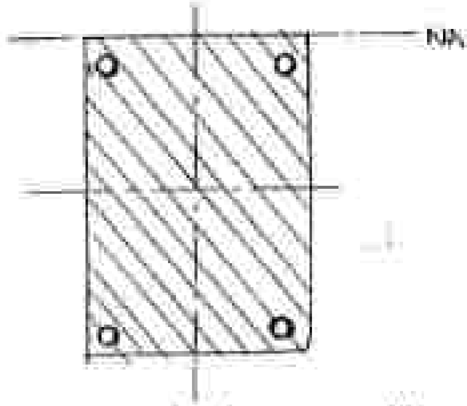
(Axial Load + Uni-axial BM).

$$\boxed{\epsilon_{max} = 0.0035 - 0.75\,\epsilon_{min}}$$

When $\epsilon_{min} = 0$, $\epsilon_{max} = 0.0035$ (same as a beam)



• Tension zone



• Steel column condition

(NA @ max compressed fibre).

Zone I : AB.

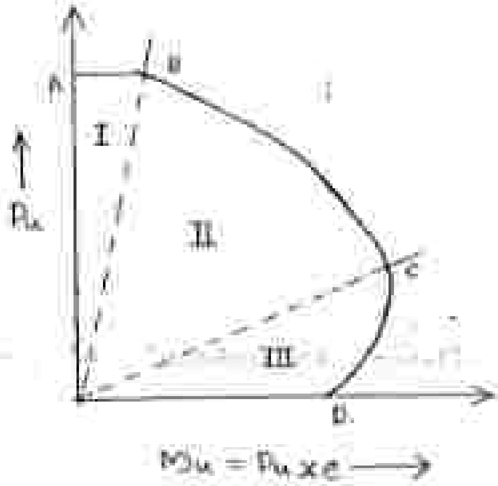
Axially loaded column with $e \leq e_{min}$.

Zone II : BC

Axial load + Uniaxial BM but column under compression

Zone III : CD

Tension zone. A part of column is in tension. Column behave like a beam; not advisable for design.



Interaction Curve (SP 16)

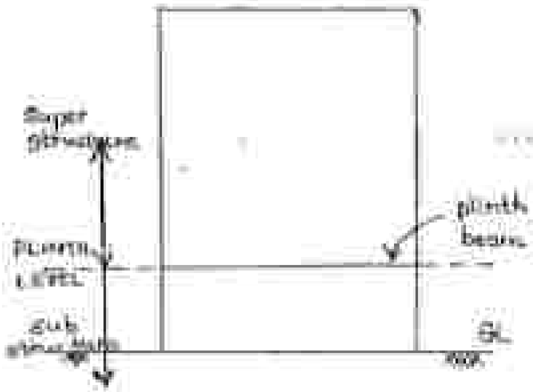
C : Balancing or limiting condition where NA touches the least compressed fibre

D : Steel Column Condition

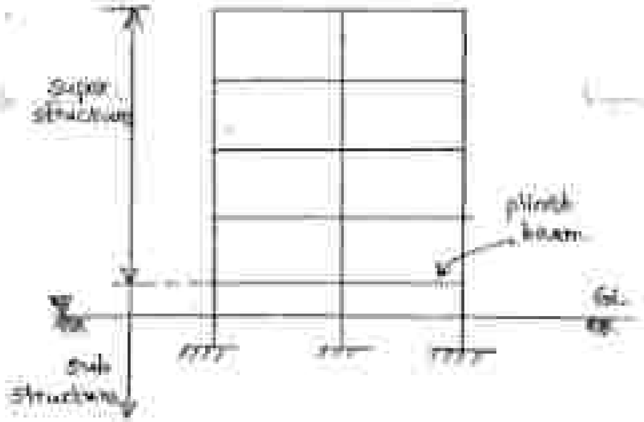
Entire column under tension and neutral axis touching max. compressed fibre; practically not possible.

6th nov,
Thursday

11. FOOTINGS



Load bearing structure



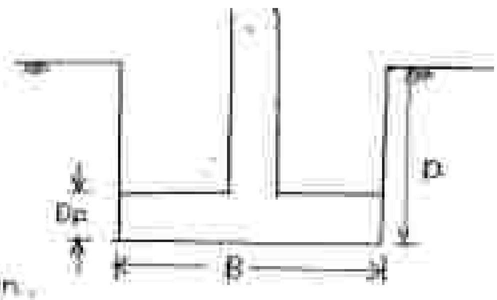
In a bridge, above bridge bearing is superstructure (deck slab).

→ Classification of Footings

According to Terzaghi,

(i) B ≮ D ⇒ Shallow (open)

(ii) B < D ⇒ deep



→ Rankine's min. depth of foundation.

$$\boxed{D_f = \frac{q_o}{\gamma}(K_a)^2}$$

q_o → pressure below footing $= \dfrac{\text{load on column}}{\text{plan area}}$

γ → weight density of soil.

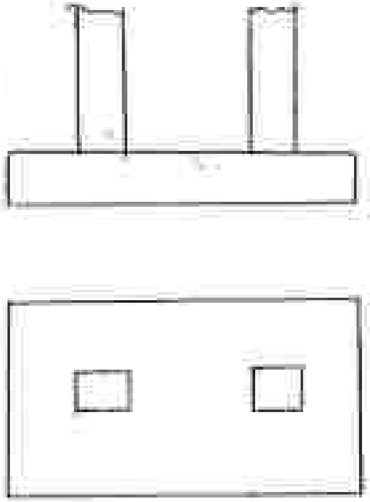
→ Shallow

(i) Isolated — one column, one footing → Rectangular

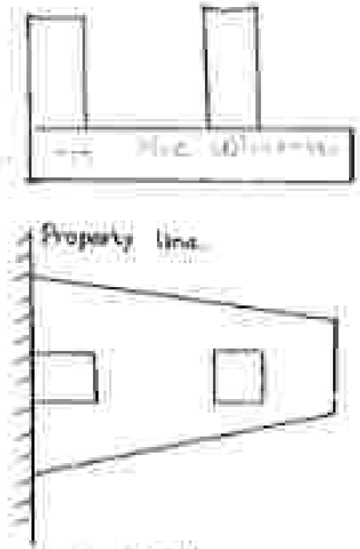
(ii) Combined — two column, one footing → Trapezoidal

(iii) Raft — all columns, one footing → Strap

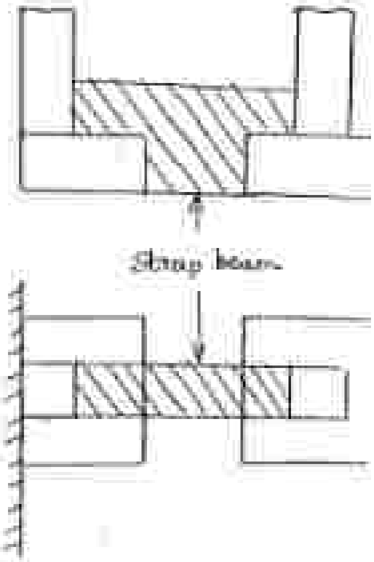
- If area of all footings exceeds 50% plinth area, then provide raft/mat foundation.

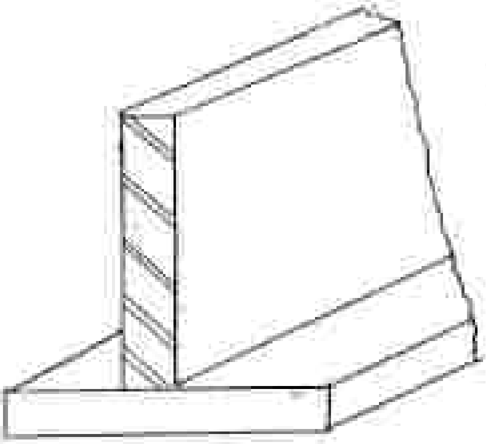


▪ combined rectangular



▪ combined trapezoidal





▪ Strip footing (one way) - used under masonry wall

Combined trapezoidal is used when a column is located on the property line.

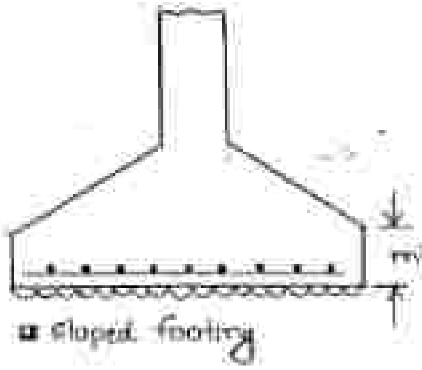
Strap beams are deep beams which are used to connect two distant columns of which one is located on the property line.

→ Deep Foundations

1. Piles - used in clay or silt
2. Well - sandy
   ↖ Caisons - pressure well.

7th nov.
Sunday → Specifications

1. Thickness at edge of footing.



(based on serviceability criteria)

2. Min % of steel.

It's also called as distribution steel, temperature steel, nominal steel. It is provided to meet the nominal requirem against secondary stresses.

MS (Fe 250) ⇒ $\underline{0.15\% \ A_g} = \frac{0.15}{100} \times bxD$

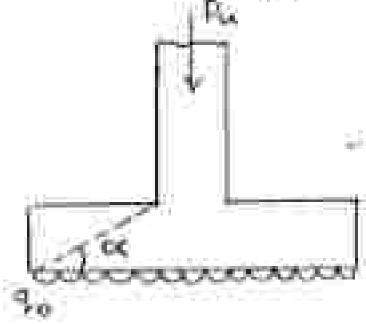
HYSD (Fe 415/Fe 500) ⇒ $\underline{0.12\% \ A_g} = \frac{0.12}{100} \times bD$

} same slab.

– min % steel is provided to distribute the load on a wider area.

3. Min clear cover = 50 mm ; for any type of exposure conditions.

4. PCC Footing (no reinforcement)



$\tan\alpha \geq 0.9\sqrt{\frac{100\,q_o}{f_{ck}}+1}$; not for GATE

$q_o = \frac{P_u}{A_f}$

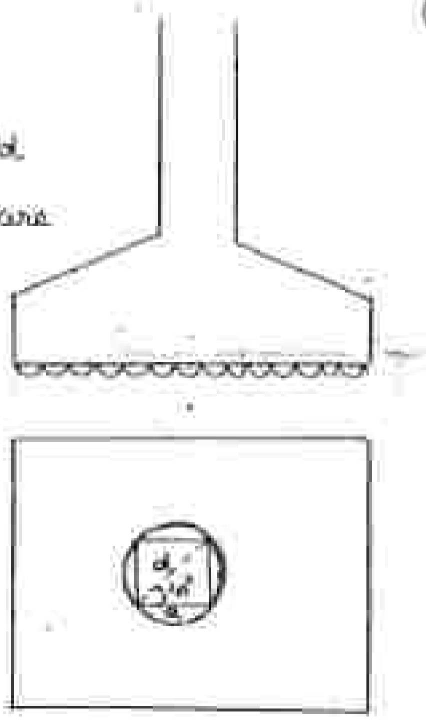
A_f → plan area of footing

NOTE:

- If a circular column is resting over the footing, the analysis should be done by considering inscribed square as shown in fig.

$$\sin 45^\circ = \frac{a}{d}$$

$$\boxed{a = \frac{d}{\sqrt{2}}}$$



→ Design

- Isolated footings design is similar to that of slabs, ie, designed for bending and checked for shear.

* BENDING MOMENT:

- Critical section for BM is at the face of column

$$\boxed{M_y = q_o A_1 \bar{x}}$$

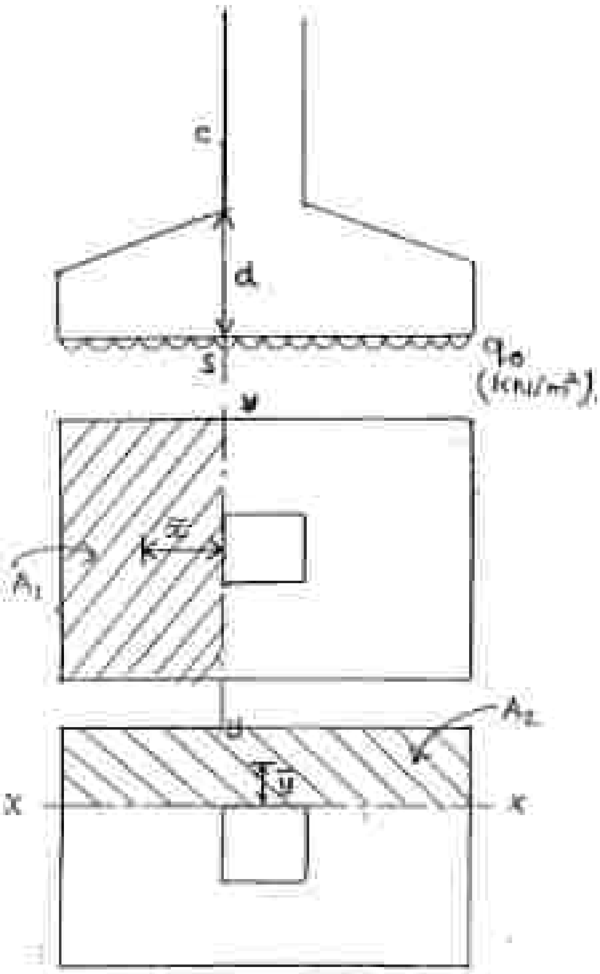
$$\boxed{M_x = q_o A_2 \bar{y}}$$

Use max of M_y & M_x for bending.

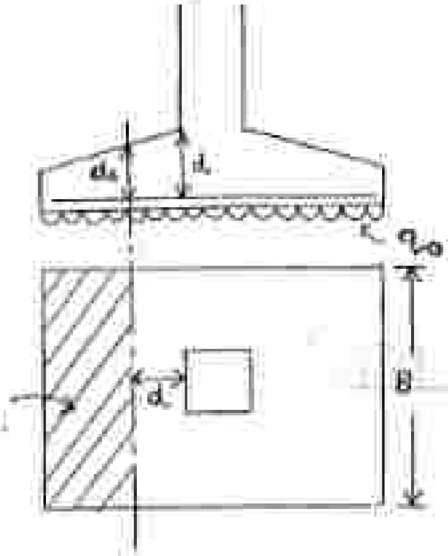
* SHEAR:

(one way / beam shear)

- critical section for one-way shear is at a distance of d from the face of column.



— Due to excessive shear, a crack along short direction develops at a distance d from face of column



One way Shear force,

$$\boxed{V_u = q_o A_3}$$

$$\boxed{\left.\begin{array}{l}\text{Nominal (average)}\\ \text{One way stress}\end{array}\right\} \tau_v = \frac{V_u}{B(d_c)}}$$

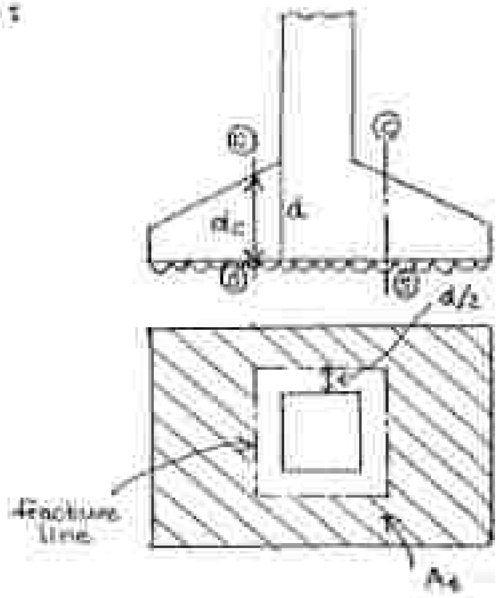
* Two Way (Punching Shear):

Two way shear force.

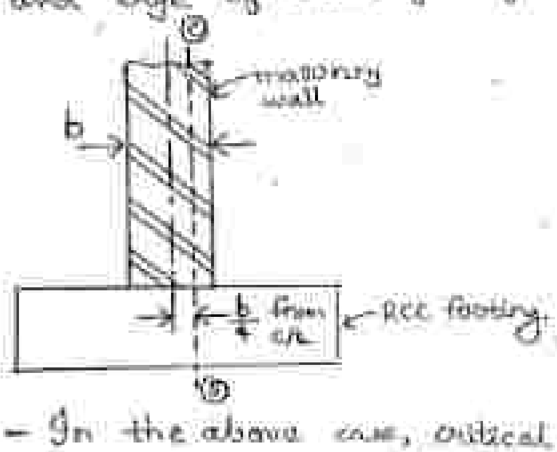
$$\boxed{V_u = q_o A_4}$$

$$\boxed{\left.\begin{array}{l}\text{Nominal Shear}\\ \text{Stress}\end{array}\right\} \tau_v = \frac{V_u}{P(d_c)}}$$

where P → perimeter of fracture line.

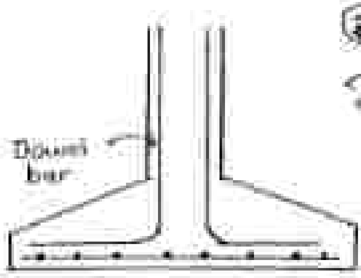


* Critical section for BM is half way b/w centre line and edge of wall, for footings under masonry walls



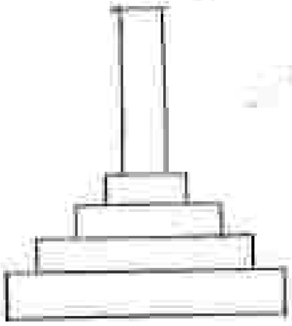
— In the above case, critical section for one way & two way shear is same as that of an RCC column resting over RCC footing

– Dowel bar is used to transfer load from column to footing.



P-58

Q.5



■ Pedestal footing.

6 Permissible bearing stress (Limit state) $= 0.45\ f_{ck}$

$= 0.45 \times 20 = \underline{9\ \text{MPa}}$

01 $q_0 = 3\ \text{MPa}$, $f_{ck} = 20\ \text{MPa}$

$$\tan\alpha \geq 0.9\sqrt{\frac{100\,q_0}{f_{ck}} + 1}$$

$$\tan\alpha \geq 0.9\sqrt{\frac{100 \times 3}{20} + 1}$$

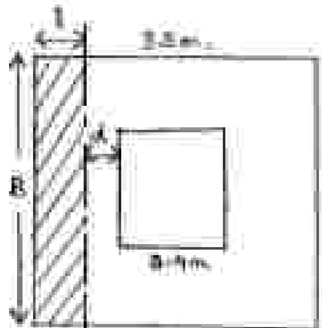
$$\underline{\tan\alpha \geq 3.6}$$

P-60

1. Pad type footing is used $\Rightarrow$ uniform thickness.

$\Rightarrow d = d_c = 0.56\ \text{m}$.

$q_0 = 122.4\ \text{kPa}$.



For one way shear,

$$l = \left(\frac{B - 0.4}{2}\right) - d$$

$$= \frac{3.5 - 0.4}{2} - 0.56 = 0.99.$$

$$V_u = q_o \times A = 122.4 \times 0.99 \times 3.5$$

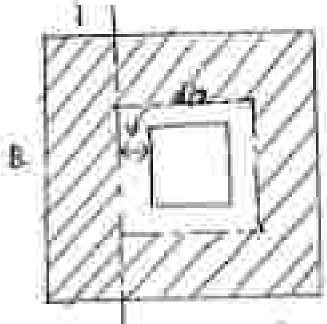
$$= \underline{424.116} \text{ kN}$$

$$\tau_v = \frac{V_u}{B \cdot d_c} = \frac{424.116}{3.5 \times 0.56} = 216.37 \text{ kPa}$$

$$= \underline{0.22 \text{ MPa}}$$

2. $\cancel{1 - \left(\frac{3.5 - 0.4}{2} - \frac{d}{2}\right)}$; ~~two way shear~~

$\cancel{= 1.27 \text{ m}}$



Two-way Shear

$\cancel{V_u = 122.4 \times 1.27 \times 3.5} =$

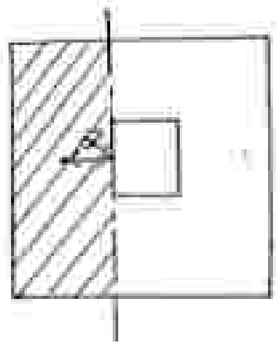
$$A = 3.5^2 - (0.4 + 0.56)^2 = 11.3284 \text{ m}^2$$

$$V_u = q_o A = 122.4 \times 11.3284 = \underline{1386.596} \text{ kN}$$

$$\tau_v = \frac{V_u}{P \cdot d_c}$$

$$= \frac{1386.596}{4(0.4 + 0.56) \times 0.56} = 644.8 \text{ kPa} = \underline{0.644} \text{ MPa}$$

3. $\bar{x} = \left(\frac{3.5 - 0.4}{2}\right)\frac{1}{2} = 0.775$



$$M_x = M_y = q_o A \bar{x}$$

$$= 122.4 \times (3.5 \times 1.55) \times 0.775$$

$$= \underline{514.615} \text{ kNm}$$

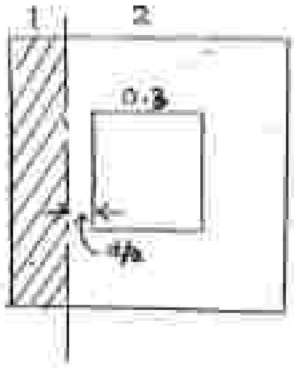
05 $P = 320$ kN

$$q_o = \frac{P}{\text{area of footing}} = \frac{320}{2 \times 2} = \underline{80 \text{ kN/m}^2}$$

38

$$l = \left(\frac{2-0.2}{2} - 0.2\right)$$

$$= 0.65 \text{ m.}$$



$$V_u = q_{u} \times A$$

$$= 80 \times 2 \times 0.65 = 104 \text{ kN.}$$

$$\tau_v = \frac{V_u}{B \cdot d_c} = \frac{104}{2 \times 0.2} = 260 \text{ kPa}$$

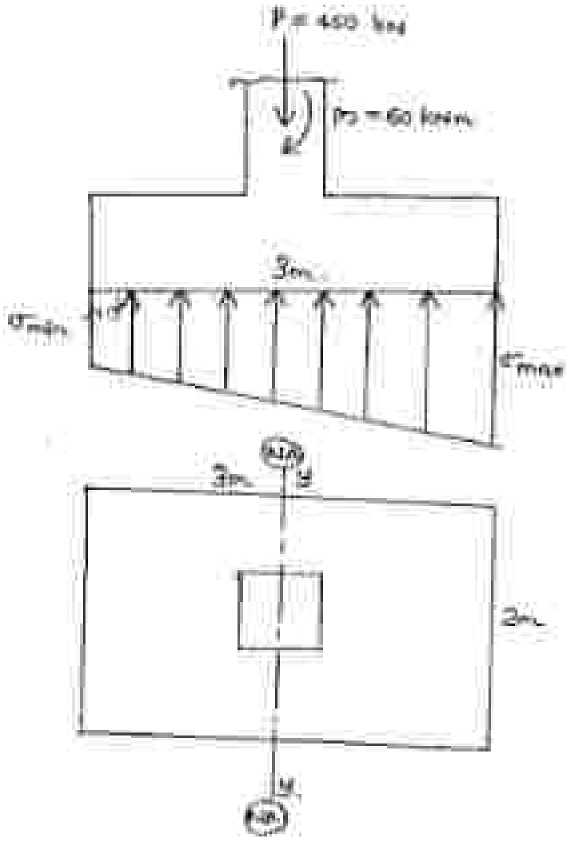
$$= \underline{0.26 \text{ MPa}}$$

04. $$\sigma_{max/min} = \frac{P}{A} \pm \frac{M}{Z}$$

$$\sigma_{max/min} = \frac{450}{2 \times 3} \pm \frac{60 \text{ kNm}}{\frac{2 \times 3^2}{6}}$$

$$\Rightarrow \sigma_{max} = 95 \text{ kPa}$$

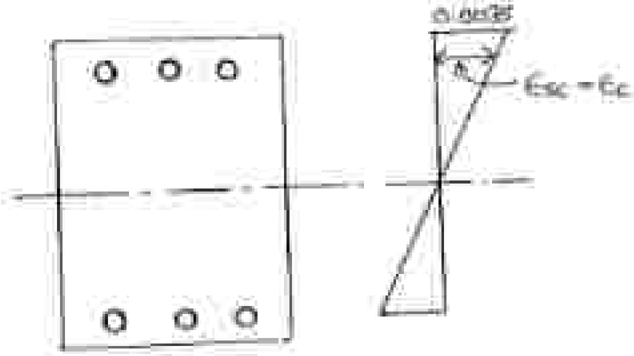
$$\sigma_{min} = \underline{55} \text{ kPa}$$



7th nov,
FRIDAY

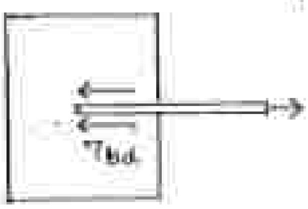
07. BOND

The bond is based on strain compatibility at a horizontal layer in a beam. Strain in concrete & steel would be same. Such a variation is valid upto failure of the beam.



→ Bond Stress: (τ_{bd})

Bond stress is the shear stress developed between steel bar and the surrounding concrete.



τ_{bd} depends on :-

(i) Grade of concrete (f_{ck})

(ii) Type of reinforcement (MS or HYSD)

(iii) Type of force in bar (tension / compression).

MS (Plain & round)

⊙ Fe 250

HYSD (ribbed or corrugated)

⊙ Fe 415

⊙ Fe 500

NOTE:

⊙ In case of Tor steel (HYSD), use $1.6\ \tau_{bd}$ (60% ↑)

⊙ In case of compression, use $1.25\ \tau_{bd}$ (25% ↑)

Compared to steel bar in tension, the bar in compression will have more bond due to poisson's effect.

- If HYSD in compression, use $2\tau_{bd}$ (100% ↑).

⇒ Plain MS in tension ⟶ τ_{bd}

Plain MS in compression ⟶ $1.25\,\tau_{bd}$

HYSD in tension ⟶ $1.6\,\tau_{bd}$

HYSD in compression ⟶ $2\,\tau_{bd}$

→ Factors affecting Bond

1. Pure Adhesion
2. Frictional Resistance
3. Mechanical Resistance (only in HYSD, due to corrugations)

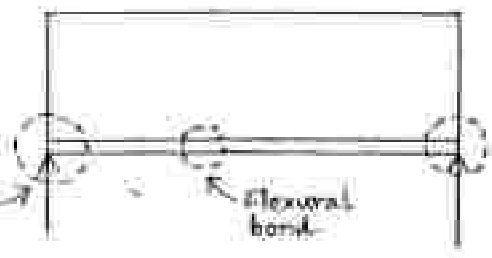
Strongest is mechanical resistance and weakest is adhesive.

→ Types of Bonds.

- Critical bond is end anchorage, then flexural

(i) End Anchorage Bond

(ii) Flexural Bond



→ Development Length (l_d)

Minimum length of embedment of steel bar in a concrete block, so that the bond between steel and concrete can resist the bar not to come out.

- Max pull that can be applied on bar, $P = \sigma_s A_s$

For equilibrium,

Max. applied force = Bond resistance

$$\sigma_s \left(\frac{\pi}{4}\phi^2\right) = \tau_{bd} * \text{surface area of embedment}$$

$$\sigma_s \left(\frac{\pi}{4}\phi^2\right) = \tau_{bd} * (\pi \phi\, l_d)$$

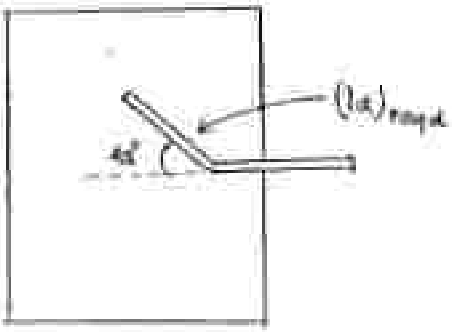
$$\boxed{l_d = \frac{\phi\,\sigma_s}{4\,\tau_{bd}}}$$

→ Anchorage values of Bends & Hooks

(i) Angle of bending = 45°

Anchorage value, AV = 4ϕ

$$\boxed{(l_d)_{req} = (l_d)_{cal} - A.V.}$$



(ii) Angle of bending = 90°

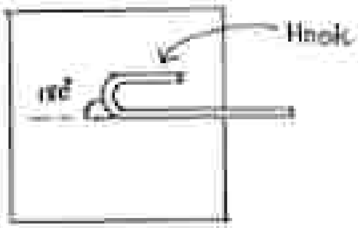
Anchorage value, AV = 2 × 4ϕ

= 8ϕ

(iii) Angle of bending = 135°

Anchorage value, AV = 3 × 4ϕ

= 12ϕ



(iv) Angle of bending = 180°

= 4 × 4ϕ = 16ϕ

– More than 180° bending is not allowed in concrete. (180° hook is possible only with MS). For HYSD, maximum allowed is 135°.

Q. A plain MS bar with bar diameter ϕ is embedded in M20 concrete, with τ_{bd} 1.2 MPa. Determine minimum length of embedment, if the bar is (i) straight in tension

(ii) in tension with hook.

(iii) in compression with 90° bend

(i) $l_d = \dfrac{\phi \sigma_s}{4 \tau_{bd}}$

$\sigma_s = 0.87 f_y = 0.87 \times 250 = 217.5$

$l_d = \dfrac{\phi \times 217.5}{4 \times 1.2} = 45.3125 \phi$

(ii) $(l_d)_{req} = (l_d)_{st} - 16\phi = (45.3125 - 16)\phi$ $\{\text{hook} \Rightarrow AV = 16\phi\}$
$= 29.3125 \phi$

(iii) Compression $\Rightarrow 1.25 \tau_{bd}$.

$l_d = \dfrac{\phi \times 217.5}{4 \times 1.25 \times 1.2} = 36.25 \phi$

$(l_d)_{req} = 36.25 \phi - 8\phi = \underline{\underline{28.25 \phi}}$ $\{90^\circ \Rightarrow AV = 8\phi\}$.

Q HYSD bar is embedded in M30 grade concrete with $\tau_{bd} = 1.5$ MPa. Determine development length of the bar is
(i) Fe 415 grade with 135° bend in tension.
(ii) Fe 500 grade with 45° bend in compression.

(i) HYSD bar $\Rightarrow 1.6 \tau_{bd} = 2.4$ MPa

$l_d = \dfrac{\phi \sigma_s}{4 \tau_{bd}} = \dfrac{\phi \times 0.87 \times 415}{4 \times 2.4} = 37.609 \phi$

$(l_d)_{req} = 37.609 - 12\phi$ (135° bend).
$= \underline{\underline{25.609 \phi}}$

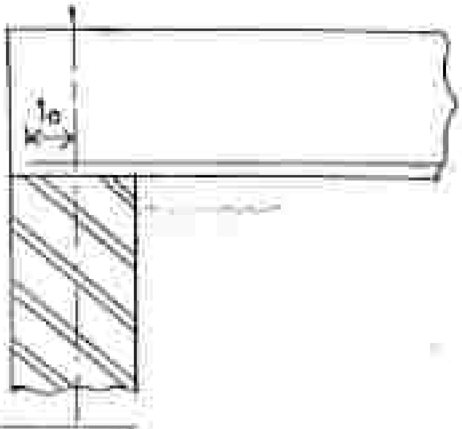
(ii) Compression $\Rightarrow 2 \tau_{bd} = 3$ MPa

$l_d = \dfrac{\phi \sigma_s}{4 \tau_{bd}} = \dfrac{\phi \times 0.87 \times 500}{4 \times 3} = 36.25$ MPa ϕ

$(l_d)_{req} = 36.25 \phi - 4\phi = \underline{\underline{32.25 \phi}}$ (45° bend).

→ Anchorage Length of Main Steel. (l_o)

The minimum extension of steel bar in a structural member beyond theoretical cut off point.



* Check for Development length.

$$\boxed{l_d \leq \frac{M_1}{V} + l_o \quad (\text{fix/continuous})}$$

$$\boxed{l_d \leq \frac{1.3 M_1}{V} + l_o \quad (\text{simply supported beam})}$$

where M_1 → moment of resistance of the beam c/s at a place where the band is to be checked.

V → V_u, shear force due to external loads.

- In case of non flexural or bending member like a column where moment of resistance, $M_1 = 0 \Rightarrow l_d = l_o$

- In flexural members of beams, $l_o < l_d$

Imp
P-37

08. $A_{st} = 2 \times \frac{\pi}{4} \times 16^2 = 402.124$

$x_{umax} = 0.48d = 0.48 \times 425$
$= 204$ mm.

$0.36 f_{ck} \, b x_u = 0.87 f_y A_{st}$.

$0.36 \times 20 \times 250 \times x_u = 0.87 \times 415 \times 402.124$.

$x_u = 80.66$ mm $\Rightarrow x_u < x_{umax}$ (UR section)

$M_R = 0.87 f_y A_{st} (d - 0.42 x_u) = 0.87 f_y \times 402.124 (425 - 0.42 \times 80.66)$
$= \overset{56.78}{\cancel{4918.48}}$ kNm.

$M_1 = M_R = \overset{56.78}{\cancel{4918.48}}$ kNm.

$$l_d = \frac{\phi \sigma_s}{4\tau_{bd}} = \frac{16 \times 0.87 \times 415}{4 \times 1.6 \times 1.2} = 752.1875$$

$(l_d)_{req} = l_d - 8\phi$. (90° bend).

$= 752.1875 - 8 \times 16 = \underline{624.2}$ mm

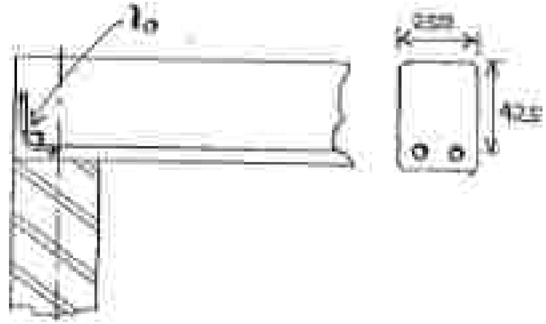
V = 220 kN

For SSB,

$$l_d \leq \frac{1.3 M_1}{V} + l_0$$

$$\Rightarrow l_0 = 0.6242 - \frac{1.3 \times 56.72}{220}$$

$$= 0.2885 \text{ m} = \underline{288.5} \text{ mm}$$



→ Flexural Bond

The safety of the flexural bond can be checked as per the above relation b/w l_d & l_0.

- For flexural bond, l_0 is unknown as the bar is extended on either side at a section under consideration. ∴ as per IS 456, rule l_0 as:

$$l_0 = \text{maximum of} \begin{cases} 12\phi \\ d \end{cases}$$

$$l_d \leq \frac{M_1}{V} + l_0 \quad ; \text{ Fix / continuous}$$

$$l_d \leq \frac{1.3 M_1}{V} + l_0 \quad ; \text{ SSB}$$

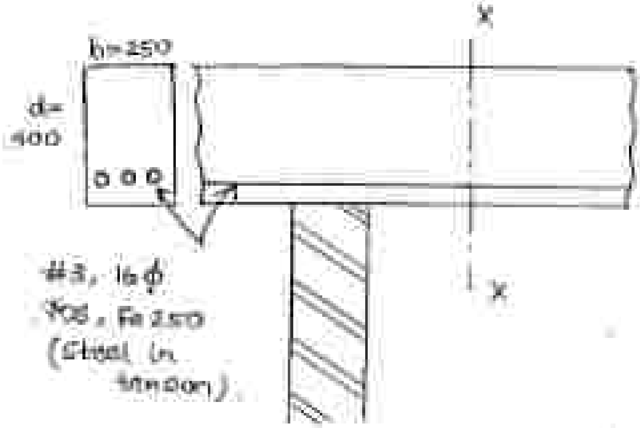
- If flexural bond is not safe, then provide <u>smaller diameter</u> bars <u>more in number</u> so that force on each bar can be reduced and the surface area of contact b/w steel and concrete increase.

Q1 $V = 150$ kN

$$(l_d)_{st} = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

$$= \frac{16 \times 0.87 \times 250}{4 \times 1}$$

$$= \underline{870 \text{ mm}}$$



$x_{umax} = 0.53 \times 400 = 212$

$$x_u = \frac{0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 15 \times 250} = 97.18 \text{ mm}$$

$x_u < x_{umax} \Rightarrow$ URS.

$M_1 = M_u = 0.36 \times 15 \times 250 \times 97.18 (400 - 0.42 \times 97.18)$
$= 47.122$ kNm.

$$l_0 = \max \begin{cases} 12\phi = 12 \times 16 = 192 \\ d = 400 \end{cases} = 400 \text{ mm} \quad \text{(Flexural bond)}$$

$$\frac{M_1}{V} + l_0 = \frac{47.122}{150} + 0.4 = 0.714 = 714 \text{ mm} < 870$$

$$\Rightarrow l_d > \frac{M_1}{V} + l_0 \text{ ; unsafe in bond.}$$

- Using additional anchorage (or) additional extension of the bar is possible only in case of end anchorage bonds.
- For flexural bonds, the alternative to make it safe is by providing smaller diameter more in number.

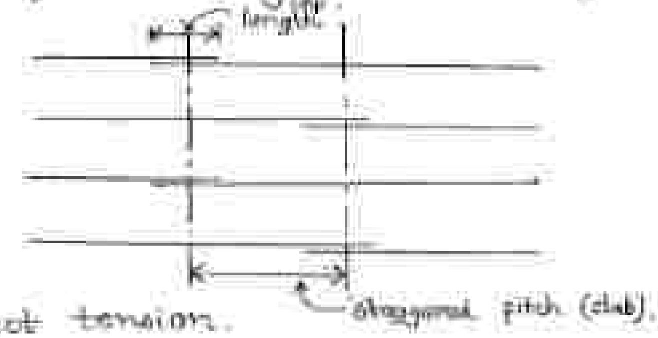
→ Splicing of Bars

- attachment of reinforcement ($l_{req} > l_{available}$)
- to change diameter (in columns).

- Max length of bar in market
  - 12 m – upto 25 mm ϕ
  - 6 m – greater than 25 mm ϕ

▫ Splice / Joint

- Splicing of bars should be avoided at a section where BM is 50% of moment of resistance of section.
- not more than 50% of bars should be spliced at a section
- Bars of diameter greater than 36 mm should be welded.



- Direct tension.

$$\left.\begin{array}{l} \text{lap} \nless 2\,l_d \\ \nless 30\,\phi \end{array}\right\} \text{use max.}$$

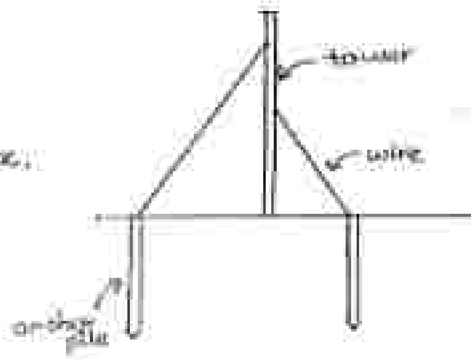
Eg. ▫ Side walls of annular water tank will be subjected to hoop or direct tension.

▫ Steel in anchor piles will be under direct tension.

- Bending Tension

$$\left.\begin{array}{l} \text{lap} \nless l_d \\ \nless 30\,\phi \end{array}\right\} \text{use max.}$$

Eg: Beam.

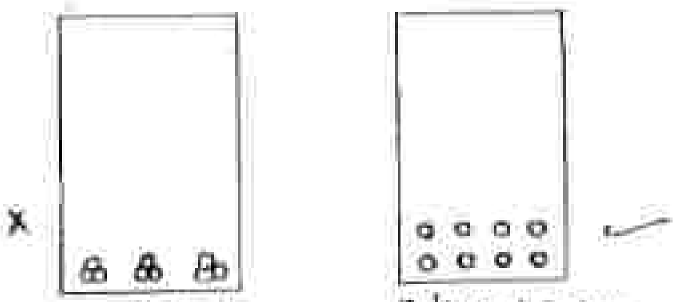


— Direct Compression (or) Bending Compression

$$\left.\begin{array}{l} l_{ap} \not< l_d \\ \quad\ \not< 24\phi \end{array}\right\} \text{use max.}$$

Axially loaded columns will be under direct compression. Compression steel in beams will be under bending compression.

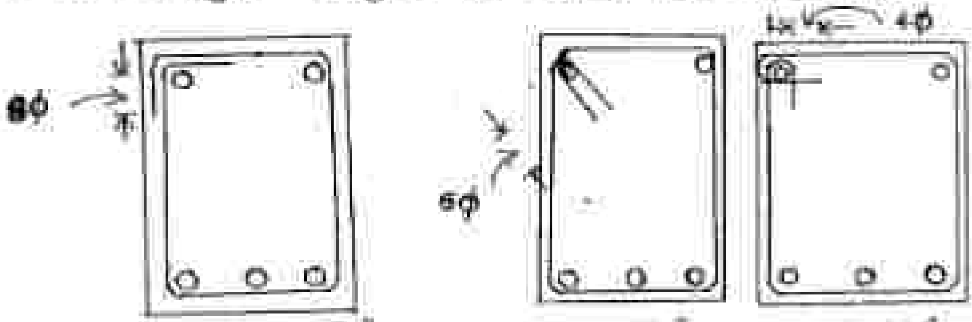
* Bundled Bars



▪ Bundled Bar (less bond)

▪ layered system

* Anchorage Length of Shear Reinforcement:



▪ 90° ▪ 135° ▪ 180°

⊙ In earthquake critical zones, <u>minimum 135°</u> of bending in stirrups is a must.

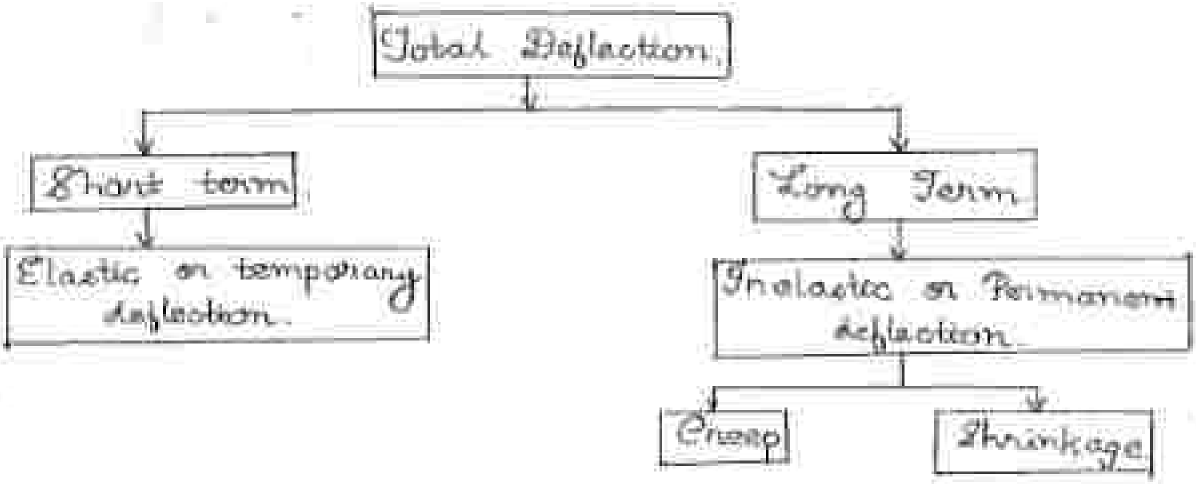
⊙ If two different ϕ bars are spliced, the lap length should be based on smaller diameter bar only (at the point of splice, larger diameter is no longer required

7th nov,
Friday

12. LIMIT STATE OF SERVICEABILITY

1. Deflections
2. Cracks
3. Vibration
4. Fire resistance
5. Durability

- If deflections are controlled, the other serviceability factors will come under control. The total deflection in a member divided into two:-

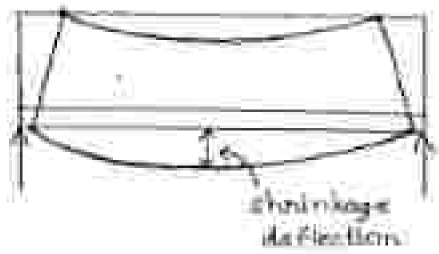


-Elastic deflections are due to live load (temporary load which can be calculated by any elastic formulae using short term modulus of concrete, ie, $5000\sqrt{fck}$)

-Creep is due to sustained or permanent loads; mainly dead load and permanent live load.

- Shrinkage occurs due to evaporation of moisture in the concrete.

– The differential shrinkage in a RCC beam causes deflection due to shrinkage.



– Most critical deflection in a beam is due to shrinkage and then creep.

– By providing doubly reinforced beams, shrinkage deflection can be reduced. To minimise shrinkage deflection, provide

$A_{sc} = A_{st}$ (which is practically impossible).

– Shrinkage and creep deflections can be calculated by empirical formulas given in IS 456 using long term modulus.

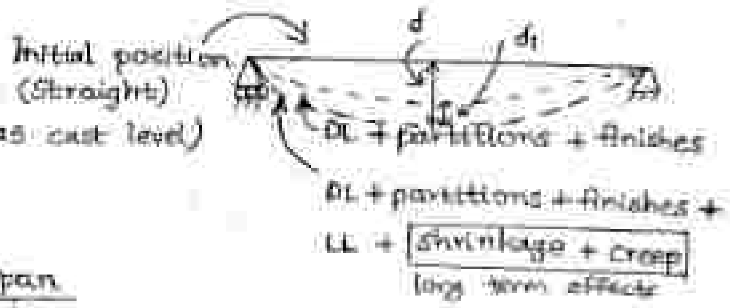
$$E_{ce} = \frac{E_c}{1+\theta}$$

where $E_c \rightarrow 5000\sqrt{f_{ck}}$

$\theta \rightarrow$ creep coefficient depending on age of concrete.

⊛ Modulus of elasticity of concrete is not a constant. It changes with grade of concrete and age of concrete, whereas for steel it is independent of grade of steel.

8th nov, Saturday → Deflection Limits



$$\left.\begin{array}{l}\text{Total deflection}\\ \text{due to all loads}\\ \text{(short + long)}\end{array}\right\} d \ngtr \frac{\text{span}}{250}$$

$$\left.\begin{array}{l}\text{Deflection after}\\ \text{DL + partitions + FF}\end{array}\right\} d_1 \ngtr \frac{\text{span}}{350} \text{ or } 20\text{ mm (use min)}$$

Due to LL + Creep + Shrinkage

NOTE :

- If δ_1 exceeds the limit, the connecting member gets cracked. 44
- If δ exceeds the limit, the member itself gets cracked.

→ Check for Deflection (as per IS 456).

- Based on $\frac{l}{d}$ ratio

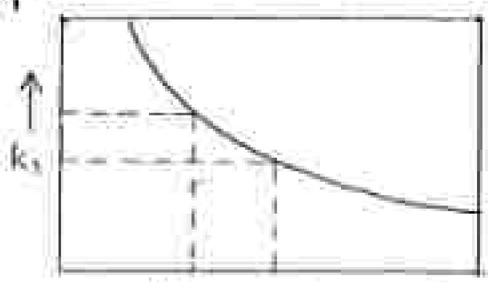
* Beams or One-way Slab :

$$\boxed{\frac{l}{d} \not> k_1 k_2 k_3 (P)}$$; P → permissible value.

Members	Span $\not>$ 10 m; P	Span > 10 m
Cantilever	7	----
Simply supported	20	$20 \times \frac{10}{\text{span}}$
Fixed / continuous	26	$26 \times \frac{10}{\text{span}}$

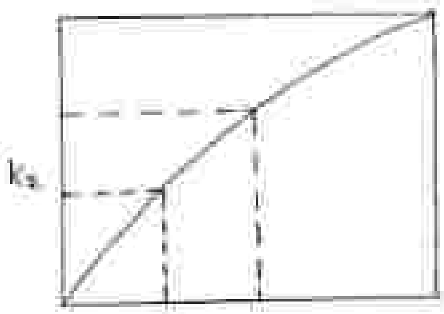
k_1 → modification factor for A_{st}

$\uparrow P_t \Rightarrow \downarrow k_1 \Rightarrow \downarrow P \Rightarrow$ actual deflection $\uparrow$



$P_t = 100 \frac{A_{st}}{bd}$ →

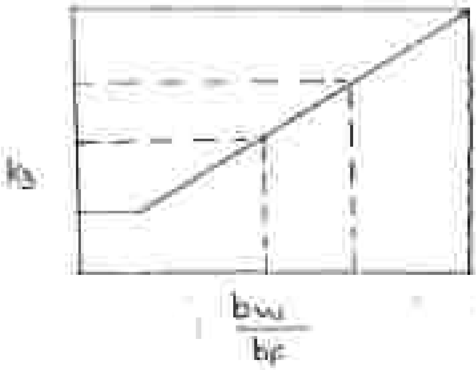
k_2 → modification factor for A_{sc}

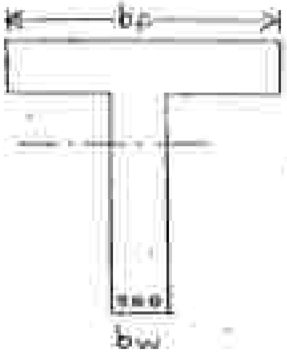


$P_c = 100 \frac{A_{sc}}{bd}$

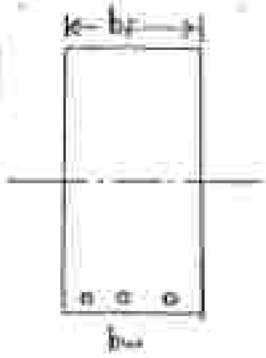
$\uparrow P_c \Rightarrow \uparrow k_2 \Rightarrow \uparrow P \Rightarrow$ actual $\downarrow$ deflection

k_3 → modification factor for flanged beam





$\frac{b_w}{b_f} < 1$



$\frac{b_w}{b_f} = 1$

$\uparrow \frac{b_w}{b_f} \Rightarrow \uparrow k_3 \Rightarrow \uparrow P \Rightarrow \downarrow$ actual deflection

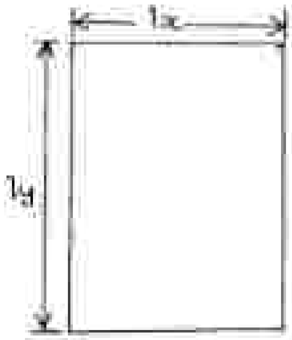
NOTE:

- Deflection point of view, rectangular beam is better compared to flanged beam. (Flanged beam will have more area of concrete in compression and more shrinkage deflection)

- Overall performance point of view, doubly reinforced flanged beam is better.

* Two-way Slab

$$\boxed{\frac{l_y}{l_x} \leq 2}$$



- live load $\leq 3\,kN/m^2$ & span (l_x) ≤ 3.5 m

↓ Member	MS	HYSD
SS	35	0.8×35
Fix/continuous	40	0.8×40

$$\boxed{\frac{l_x}{D} \leq P}$$

- In slabs, MS is better for deflections.

④ If span exceeds 3.5m or LL exceeds 3.0 kN/m², then use permissible values as per one way slab or beam. ⑩ 45

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7. $\frac{l}{d} \ngtr K_1 K_2 K_3 \, P.$

(not given)

$$\frac{4000}{d} \leq 1.1 \times 1.2 \times 1 \times 29$$

$$d = \underline{151.51 \text{ mm}}$$

11. After DL + part + finishes

$$\left.\begin{array}{l} \delta_1 \ngtr \text{span}/250 \\ 20 \text{ mm} \end{array}\right\} \text{use min.}$$

19. $\frac{15}{d} = 1 \times 1 \times 1 \times 20 \times \frac{10}{15}$

$$d = \underline{\underline{1.125 \text{ m}}}$$

17. $\delta_1 \ngtr \frac{l}{350} = \frac{10,000}{350} = \underline{28.6 \text{ mm}}$

$\ngtr 20$ mm.

10th Nov,
MONDAY

09. SLABS

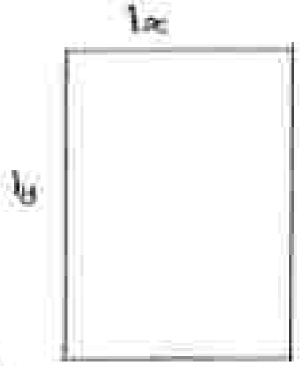
The main design criteria of a slab is deflection followed by BM & SF

→ Classification

1. Based on Aspect Ratio (l_y/l_x)

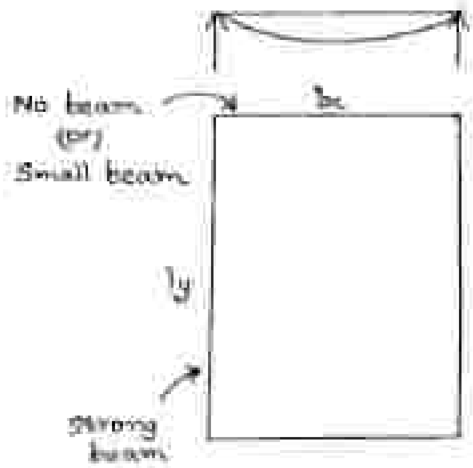
(i) $\frac{l_y}{l_x} > 2 \Rightarrow$ One way slab

(ii) $\frac{l_y}{l_x} \leq 2 \Rightarrow$ Two way slab (supported on all four edges)



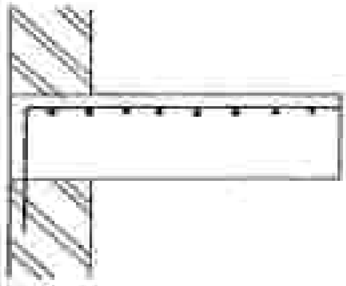
- The minimum steel is required in a slab to avoid sudden or abrupt failure of the slab.
- The maximum spacing of reinforcement is based on 'crack width criteria'

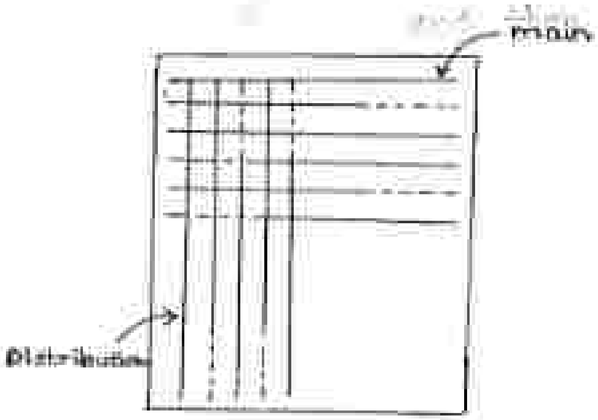
Longer direction (l_y) is restrained by providing stronger beams whereas shorter direction are supported with small beams or sometimes no beams at all.

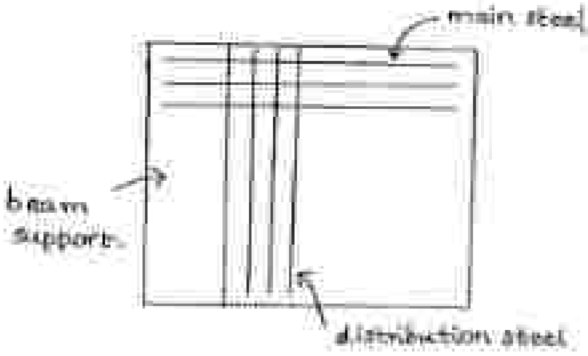


→ One way

- determinate







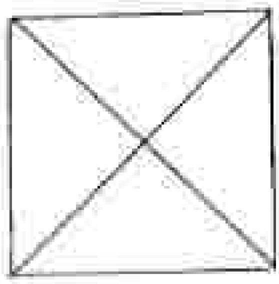
→ Two Way

- Bends in two directions.
- Indeterminate.

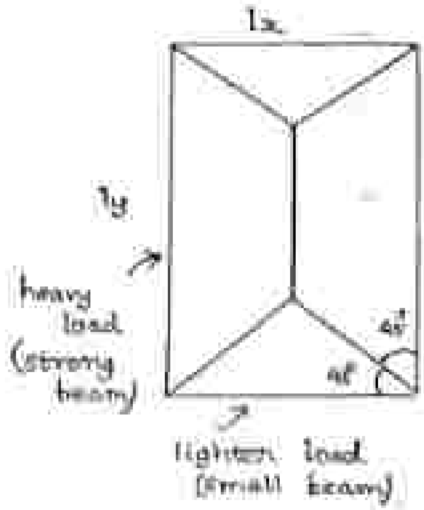
$$M_x = \alpha_x w l_x^2$$

$$M_y = \alpha_y w l_y^2$$

where α_x & α_y are bending moment coefficients.



a square slab



→ Type of two way slabs

1. Simply Supported.

- indeterminate as it bends in two directions simultaneously
- Rankine Grashoff theory used to design SS two way slabs

This theory gives α_x & α_y values, ~~based on which~~ M_x & M_y are obtained.

- In this case, corners are free to lift up. But, there is no restriction on restrainment. ∴ no torsion develops at the corner.

$$M_x = \alpha_x \, w l_x^2 \rightarrow \text{critical (max. magnitude) BM.}$$
$$M_y = \alpha_y \, w l_x^2$$

Eg: Bridge Deck slab, Slab over load bearing walls (no beams)

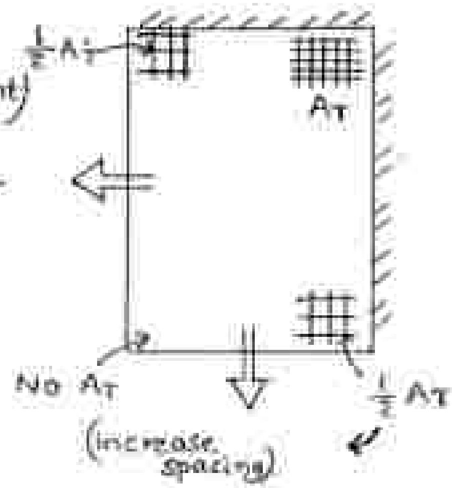
2. Restrained / Fixed or Continuous two way

- highly indeterminate.
- Jonson's Yield Line theory → α_x & α_y are given.
- In case of restrained slabs, corners wanted to lift due to bending in 2 directions, but supports are restraining the lift. ∴ torsion develops at a corner where two discontinuous edges are meeting in the form of grid or mesh.

$A_T = \frac{3}{4}$ (max. mid span reinforcement)

$A_T = 0.75 \, (A_{stx})$; in each layer of mesh.

- One mesh on top face and other on bottom face are required.



- Over a continuous edge, -ve rft is also reqd on the top face to cater hogging moment over continuous support

→ Specifications

* Max. size of coarse aggregate $= \frac{1}{4} D$
* Max size of main steel bar $= \Big\} \not> \frac{1}{8} D$

where $D \rightarrow$ total thickness of slab

Eg : $D = 150$ mm

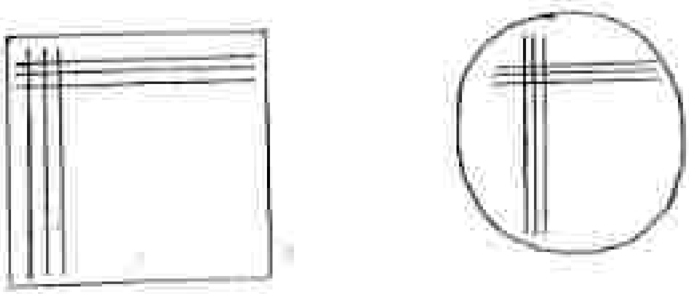
Max size of CA $= \frac{1}{4}(150) = 37.5$ mm $\approx$ 37 mm

Max. size of steel $= \frac{1}{8} 150 = 18.7$ mm $\approx$ 16 mm

→ Isotropic Slab

$$A_{stx} = A_{sty}$$

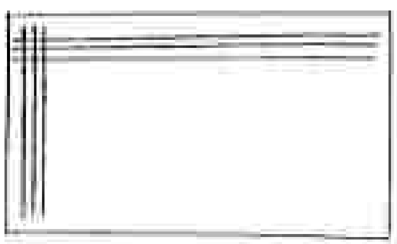
Eg: Square slab, circular slab.



→ Orthotropic Slab

$$A_{stx} \neq A_{sty}$$

Eg: rectangular slab

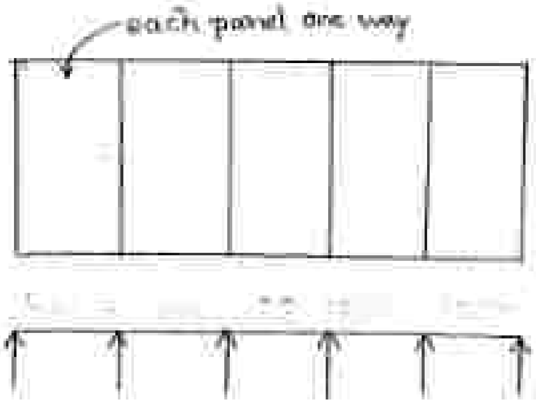


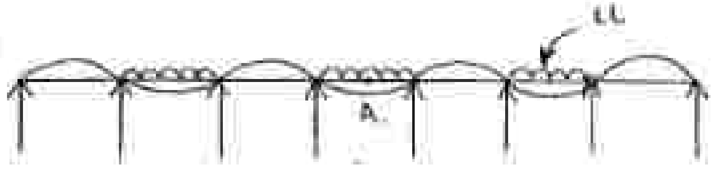
→ Continuous Slab

* Westergaard's Analysis

– Pattern Loading Method

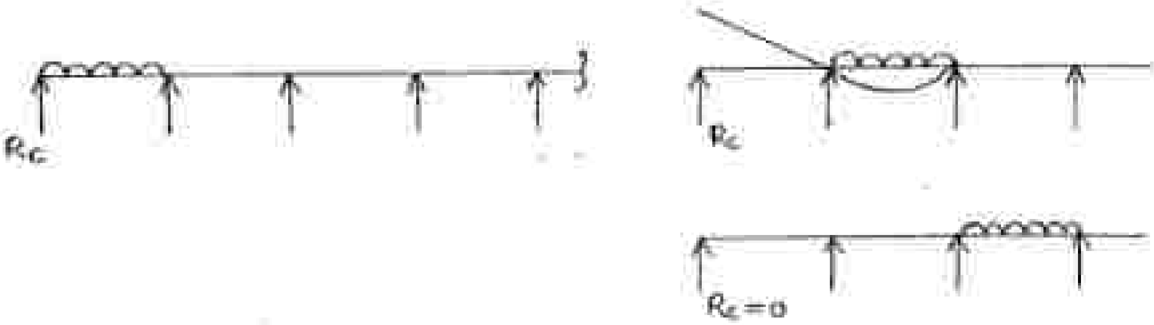
– based on Max. sagging BM (mid span @ A)





– based on maximum hogging BM (@ supports, @ B).

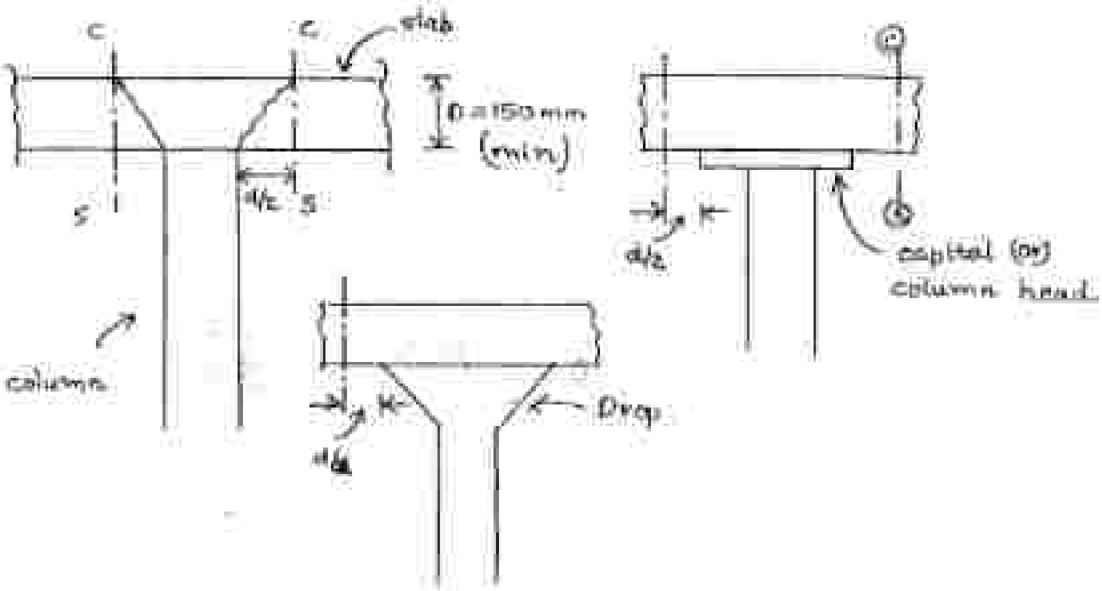
– max. SF @ end support (max reaction @ end support)



– max SF @ intermediate support (max reaction)

P-45

8.



Flat Slab :

Design criteria → Punching Shear

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PRESTRESSED CONCRETE

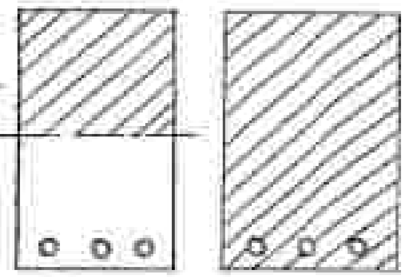
- Design based on IS 1343 - 1980

Prestressing :

- Stressing prior to loading
- entire concrete is under compression.

Steel in RCC → passive role

Steel in PSC → active role.



- RCC
- PSC

→ Material.

1. Concrete. - high grade concrete

(i) Pre-tensioning → M40

(ii) Post-tensioning → M30

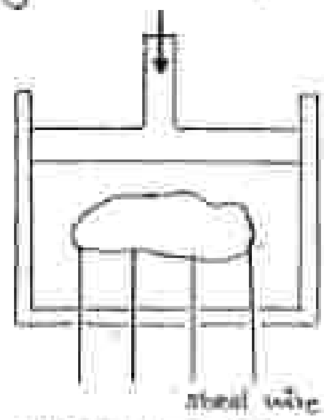
2. Steel - high tension (HT)

Eg: HT 2350

f_y = 2350 MPa. (10 times stronger than M5)

- Diameter of steel wire = 2 to 5 mm

Cable
Tendon
Strand
} wires twisted together.



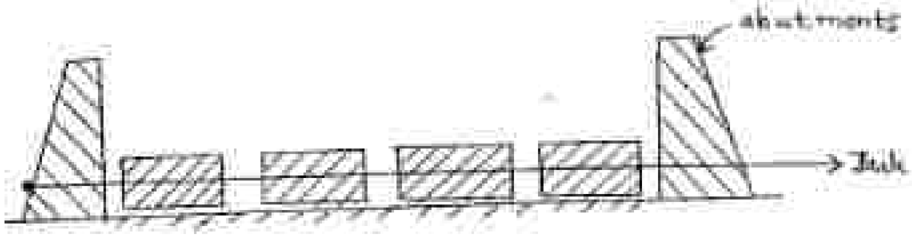
- cold process

→ Methods

1. Pretensioning

- Tensioning is done prior to casting of girder
- used in sleepers in railway lines
- 1940

* Hoyer (long line) system

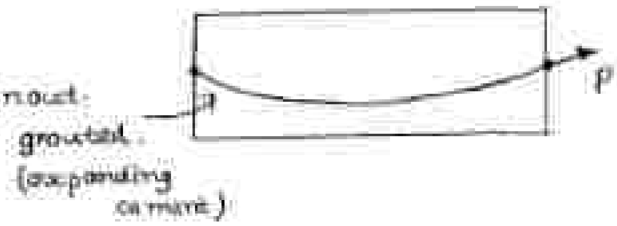


- prestress transfer : Bond or friction b/w wire and concrete.

2. Post tensioning

- Tensioning is done after casting.
- used in long span girders, bridges, buildings, roads, slabs etc.
- Prestress transfer:

End anchors + bond through grout.



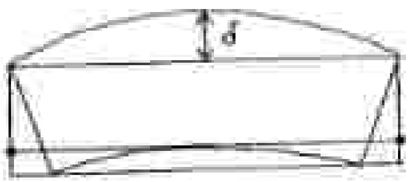
- M30

* Freyssinet Method

- popular
- 32 wires can be tensioned at a time (↓ losses)
- end anchorages : cone wedges

* Magnel System.
  - two wires can be tensioned at a time. (↑ losses)
  - end anchorages : flat plates.
* Gifford Udall System.
  - One wire at a time.
  - end anchorages : cone wedges
* Le - McCall System.
  - used to prestress wooden beams (400 years ago).
  - nut & bolt system
  - prestressing bars are used.
  - end anchorages : nut & bolt system.

- Upward deflection due to Prestress.



If $\boxed{\delta \ngtr \frac{\text{span}}{200}}$

If $\delta > \frac{\text{span}}{300}$, there will be <u>cracks on the top face</u> at the time of transfer of prestress itself.

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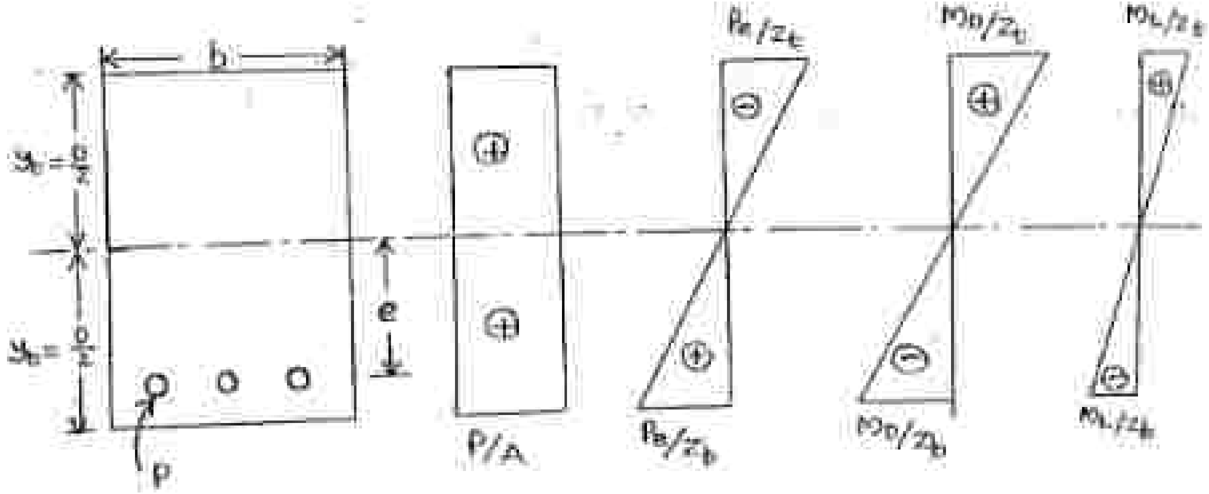
ANALYSIS OF PRESTRESSED CONCRETE



→ Resultant Stresses

1. Only due to Prestress (P)

$$\boxed{\left.\begin{matrix}\sigma_t \\ \sigma_b\end{matrix}\right\} = \frac{P}{A} \mp \frac{Pe}{Z_{t/b}}}$$

$$Z_t = \frac{I}{y_t}$$

$$Z_b = \frac{I}{y_b}.$$

For symmetric section,

$$Z_b = Z_t = Z.$$

2. Transfer / Initial condition (P + DL)

$$\boxed{\left.\begin{matrix}\sigma_t \\ \sigma_b\end{matrix}\right\} = \frac{P}{A} \pm \frac{M_D}{Z_{t/b}} \mp \frac{Pe}{Z_{t/b}}}$$

3. Service / working Condition (P + DL + LL + losses)

$$\boxed{\left.\begin{matrix}\sigma_t \\ \sigma_b\end{matrix}\right\} = \frac{\eta P}{A} \mp \frac{\eta Pe}{Z_{t/b}} \pm \frac{M_D}{Z_{t/b}} \pm \frac{M_L}{Z_{t/b}}.}$$

where $P \rightarrow$ initial prestressing force.

$\eta \rightarrow$ efficiency factor.

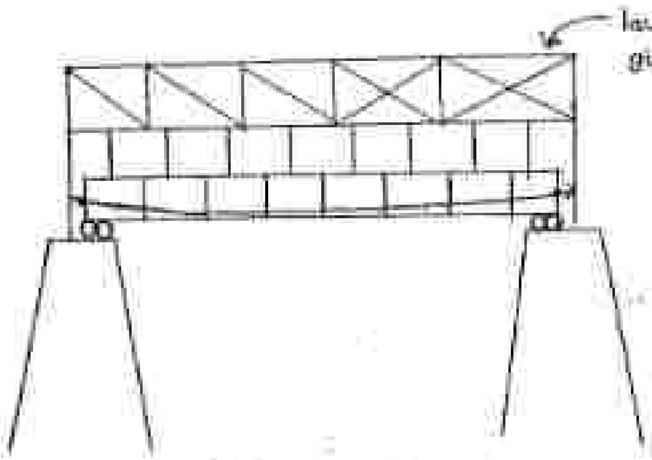
Eg: (i) For 20% loss, $\eta = 0.8$

(ii) For 30% loss, $\eta = 0.7$

$\eta P \rightarrow$ effective prestress (after loss)

NOTES:

- At transfer condition, top fibre may be subjected to tension
- At service condition, bottom fibre may be subjected to tension.



- Segmental prestress.

- Cable line (C-line):

The path along which prestressing cable is placed.

- Along the cable line, the tensile force in the steel is acting

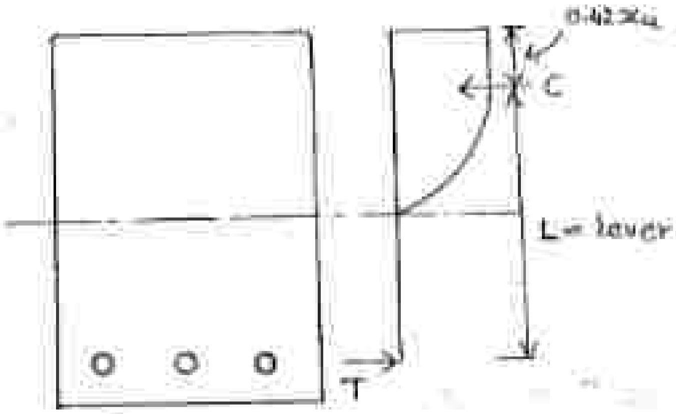
- Pressure Line (P-line) (or) Thrust line:

Imaginary line along which the resultant compressive force is acting on the girder.

- Shift of P-line from C-line: (Lever arm)

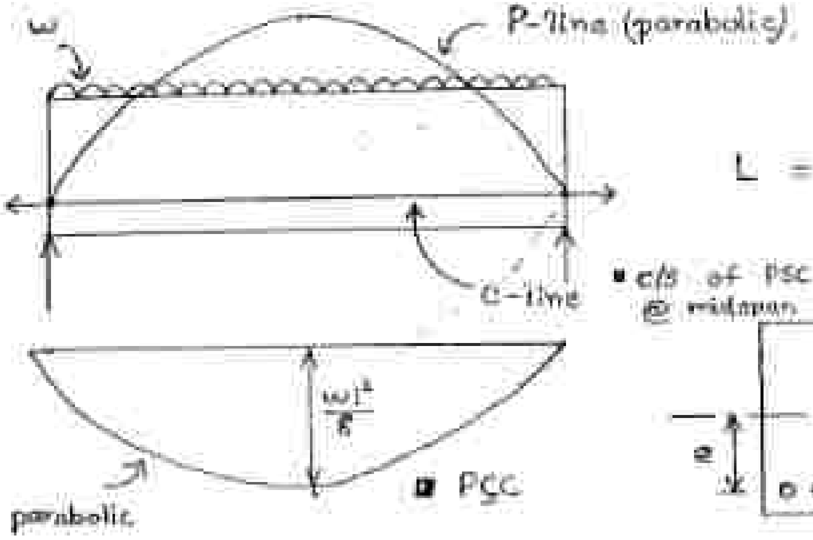
Lever arm, $L = \frac{M}{P}$; $M \rightarrow$ BM due to ext. loads

$P \rightarrow$ prestressing force in the



$$\boxed{L = d - 0.42\,x_u}$$

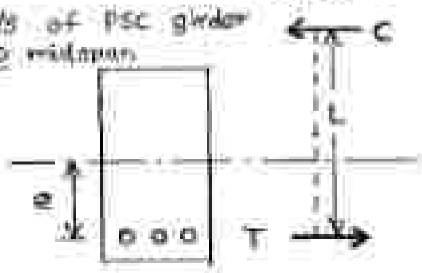
- RCC



- PSC

$$L = \frac{M}{P_e}$$ comb.

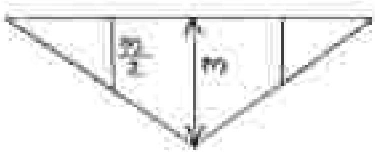
- c/s of PSC girder @ midspan



Moment of resistance, $MR = C \times L$
$= T \times L$

Q A prestressed concrete girder, 10 m span is 300 mm wide & 500 mm effective depth has a straight cable of with prestressing force of 800 kN located at 50 mm from the soffit of the beam. The beam is subj. to a central pt. load of 500 kN. Locate the P-line at the central span and also at 1/4th span.

$W = 500$ kN, $l = 10$ m, $b = 300$ mm, $d = 500$ mm.



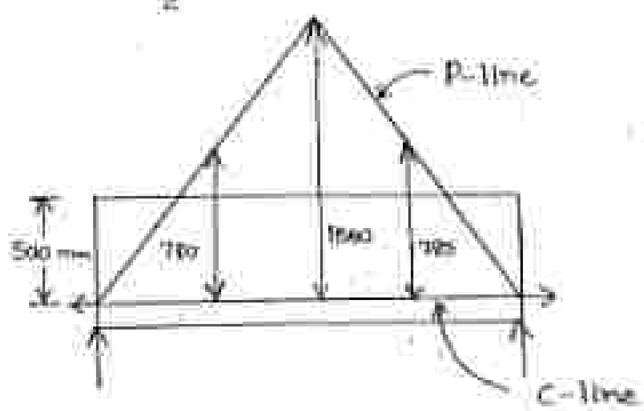
$M = \frac{wl}{4} = \frac{500 \times 10}{4} = 1250$ kN m (at central span)

$P = 800$ kN

Shift, $L = \frac{M}{P} = \frac{1250}{800} = 1.56$ m $= \underline{1560}$ mm

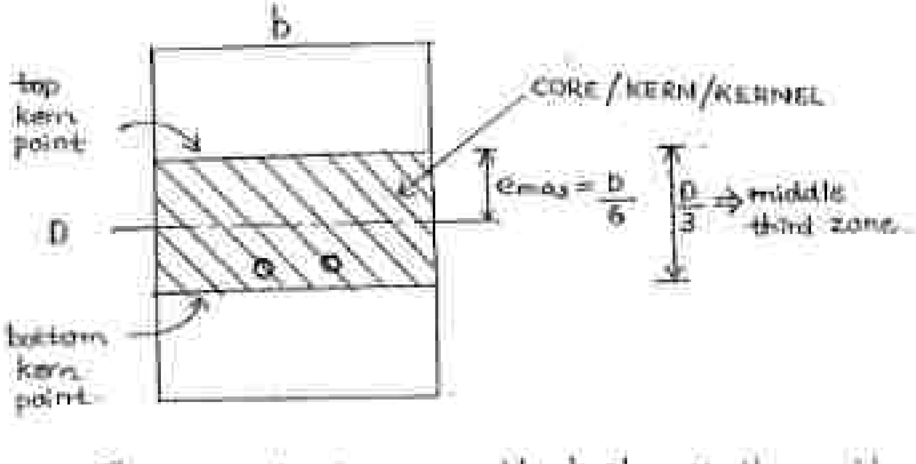
$M = \frac{1250}{2} = 625$ kN m (at $1/4^{th}$ span).

Shift, $L = \frac{1.56}{2} = 0.78$ m $= \underline{780}$ mm

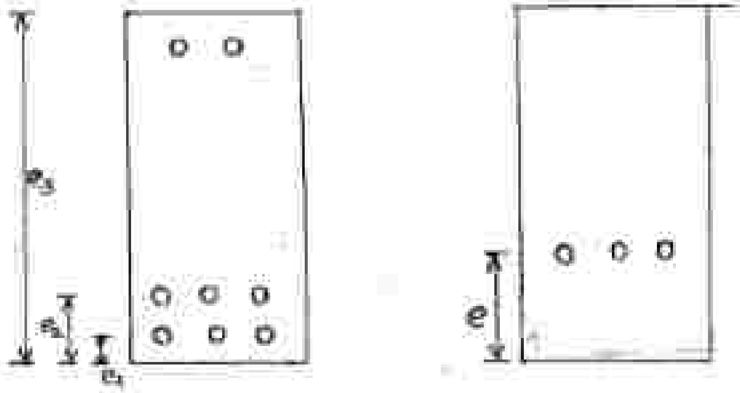


→ Limiting Cable Zone.

The zone in which resultant prestressing cable can be placed so that no tension develops in the girder under all circumstances (transfer or service condition).



⊙ For no tension, resultant of all the cables in a girder should fall in the core zone.



$$e = \frac{n_1 e_1 + n_2 e_2 + n_3 e_3}{n_1 + n_2 + n_3}$$

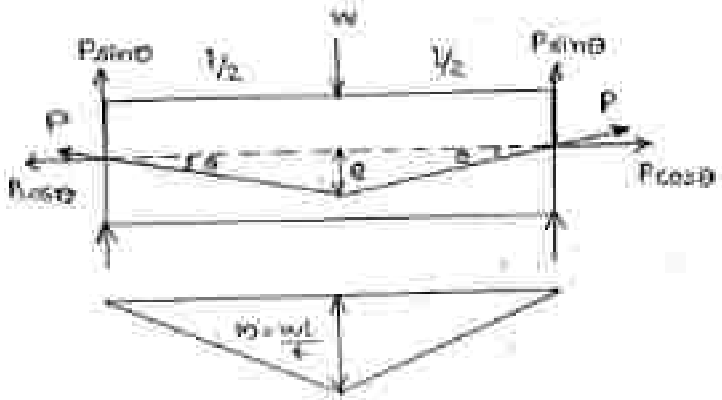
NOTE:

- If the resultant prestressing force is on the bottom Kern point, resultant stress at top extreme fibre will be zero
- If it moves below the bottom kern point, top face experiences tensile stress.
- vice-versa for top kern point.

→ Load Balancing Concept

By using suitable cable profile, external live loads can be directly balanced. → load balancing concept.

(i) concentrated load at midspan.



NOTE:

- In this concept, external L.L is balanced by vertical component of prestressing force.
- The DL will be transferred through the support in a general manner

- Here the cable profile should follow BMD and taken into tension zone.
- The horizontal component of the prestressing force to resist shear forces due to loads.

$$\Rightarrow \quad 2P \sin\theta = W \quad \rightarrow (1)$$

For smaller triangles, $\sin\theta \approx \tan\theta = \frac{e}{l/2} = \frac{2e}{l} \quad \rightarrow (2)$

$$\frac{4Pe}{l} = W \quad (\text{from } (1) \,\&\, (2)).$$

$$\boxed{P = \frac{Wl}{4e}}$$

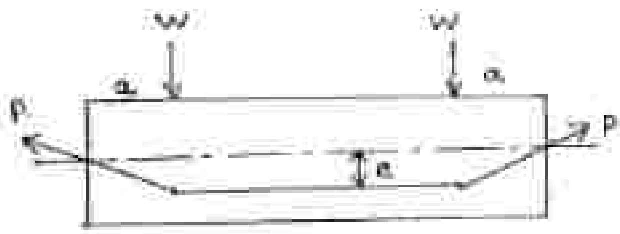
(OR)

External BM $= P.e.$

$$\frac{Wl}{4} = Pe$$

$$\Rightarrow P = \frac{Wl}{4e}$$

(ii) Two concentrated loads

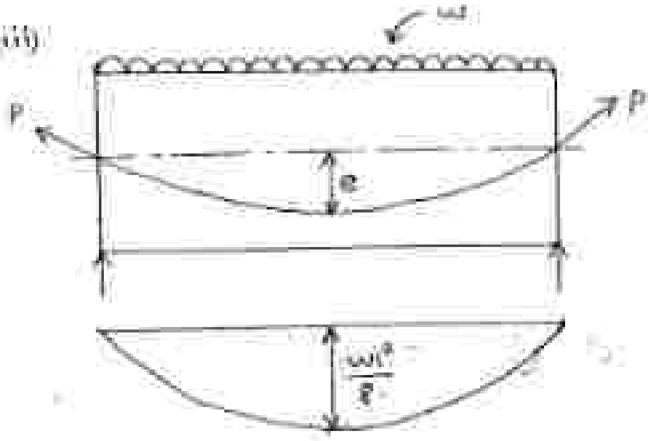


$$\Rightarrow P = \frac{Wa}{e}$$

External BM $= Pe$

$$Wa = Pe$$

(iii)

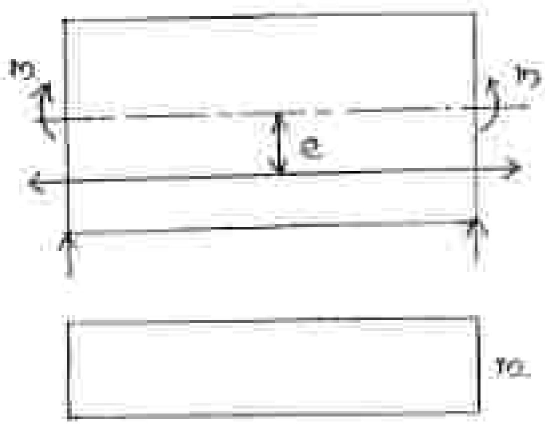


External BM = Pe.

$$\frac{wl^2}{8} = Pe$$

$$P = \frac{wl^2}{8e}.$$

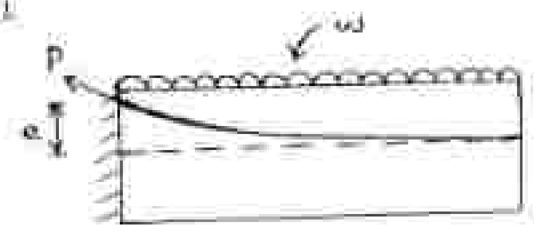
(iv) SSB subjected to moment

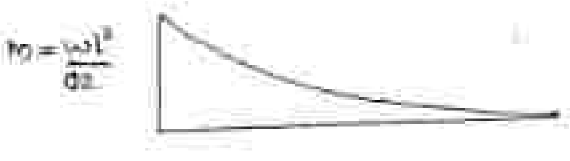


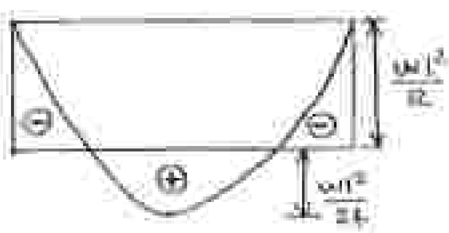
$$m = Pe$$

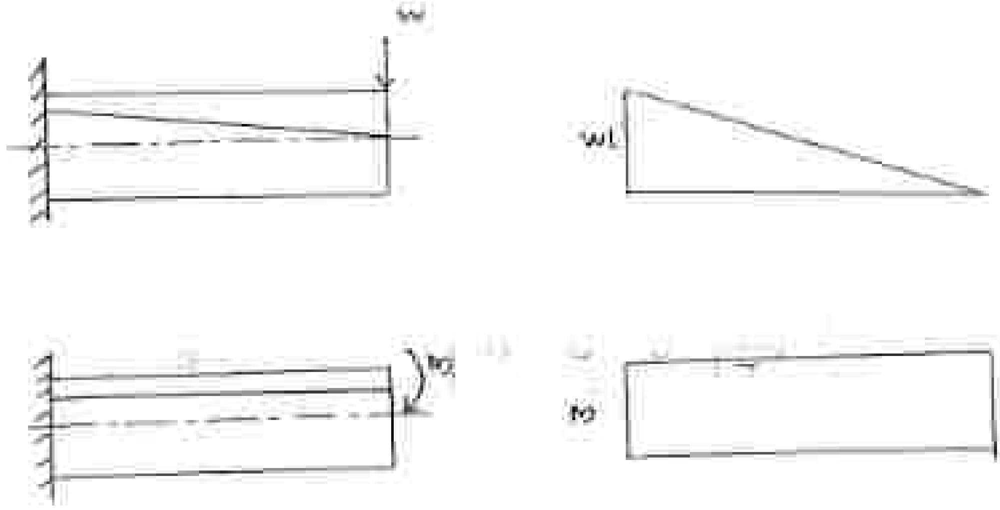
$$P = \frac{m}{e}.$$

(vi)









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2. Initial at transfer $\rightarrow$ P + DL

Top (tension), $\sigma_{top} = \frac{P}{A} \mp \frac{Pe}{z_{t/b}} \pm \frac{M_D}{z_{t/b}}$.

Bottom (compression), $\sigma_{bottom} =$ ↗

For rectangular, $z_t = z_b = bz = \frac{300 \times 600^2}{6}$

$w_D = \gamma_c \times c/s$ area.

$= 24 \times 0.3 \times 0.6 = \underline{4.32}$ kN/m

$M_D = \frac{w_D l^2}{8} = \frac{4.32 \times 6^2}{8} = 19.36$ kNm

$\sigma_{top} = -2$ MPa $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M_D}{Z} \rightarrow (1)$

$\sigma_{bottom} = +20 = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_D}{Z} \rightarrow (2)$

Solving (1) & (2).

$18 = \frac{2P}{A} \Rightarrow P = \frac{18 \times 300 \times 600}{2} = \underline{1620 \text{ kN}}$

Substituting $P = 1620$ kN in (2),

$$20 = \frac{1620 \times 10^3}{300 \times 600} + \frac{1620 \times 10^3 \times e}{300 \times \frac{600^2}{6}} - \frac{19.36 \times 10^6}{300 \times \frac{600^2}{6}}$$

$\underline{e = 135 \text{ mm}}$

Q.4. Cable line @ upper kern point (max. prestressing force).
↳ $(\sigma_R)_{bottom} = 0$

Given to use : $P + DL + LL$ (Service condition)
⇒ Assume no loss.

$$w_{DL} = \gamma_c (\text{c/s area}) = 24 \times 0.15 \times 0.3$$
$$= 1.08 \text{ kN/m}$$

$$M_D = \frac{w_D l^2}{8} = \frac{1.08 \times 10^2}{8} = 13.5 \text{ kNm}$$

$$(\sigma_R)_{bottom} = 0 = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z}$$

$$0 = \frac{500 \times 10^3}{150 \times 300} + \frac{500 \times 10^3 \times 50}{150 \times \frac{300^2}{6}} - \frac{13.5 \times 10^6}{150 \times \frac{300^2}{6}} - \frac{M_L}{150 \times \frac{300^2}{6}}$$

$M_L = 36.5$ kNm

For beam with central point load, Q

$$M_L = \frac{Ql}{4}$$

$$36.5 = \frac{Q \times 10}{4}$$

$$\Rightarrow \underline{Q = 14.6 \text{ kN}}$$

* Concordant Cable Profile

The cable profile in which p-line coincides with C-line. This profile is possible in Load balancing concept where entire LL is balanced by vertical component of prestressing force. ∴ LL will not transfer to the support.

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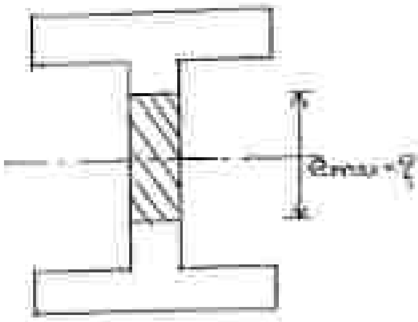
Q.16 $\sigma_t = 0$



$$\frac{P}{A} - \frac{Pe}{Z} = 0$$

$$\boxed{e = \frac{Z}{A}}$$

For I-section, Z is more than that of rectangular.

For rectangular, $e_{max} = \frac{d}{6}$

∴ For I-section, Z is more than that of rectangle.

∴ e_{max} for I section should be more than $\frac{d}{6}$

$$\frac{d}{6} = \frac{300}{6} = \underline{\underline{50\text{ mm}}}$$

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15. LOSSES IN PRESTRESSED CONCRETE

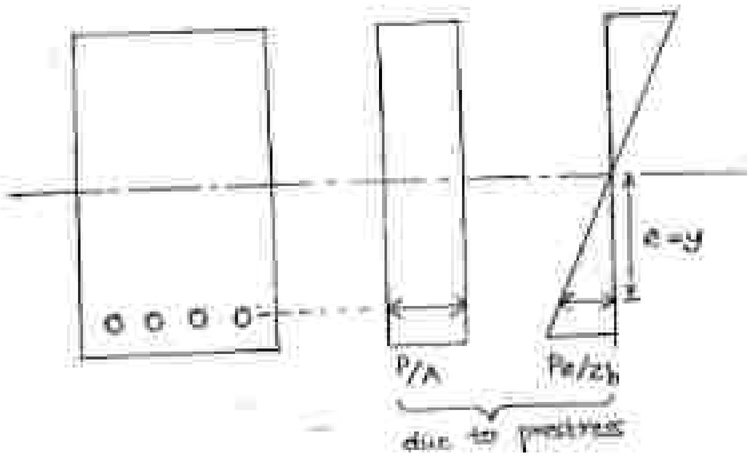
→ Loss due to Elastic Deformation (or) Shortening of Concrete

— immediate loss

Loss of prestress $= m f_c$

f_c → stress in concrete at the level of prestressing steel

■ Hoyer System



$$f_c = \frac{P}{A} + \frac{Pe}{I} y$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} (e)$$

Modular ratio, $m = \frac{E_s}{E_c}$

⊙ This loss is compulsory in all the wires in pretensioned members.

⊙ If 'n' wires are in a pretensioned girder, the total loss due to elastic shortening is $n(m f_c)$

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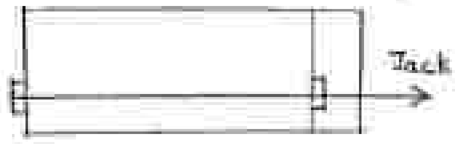
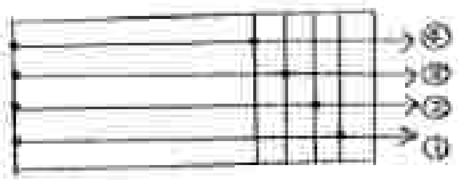
⊙ If simultaneous tensioning (all the wires are tensioned at the same time) is done, the elastic shortening of the girder occurs parallel to the tensioning process. By the time steel wires are anchored, elastic shortening process will be completed.



■ Post tensioning (Freyssinet System).

∴ loss due to elastic shortening will be zero

⊙ Due to successive tensioning (tensioning one after the other) there will be loss of prestress in the previously tensioned wire.



■ udall system.

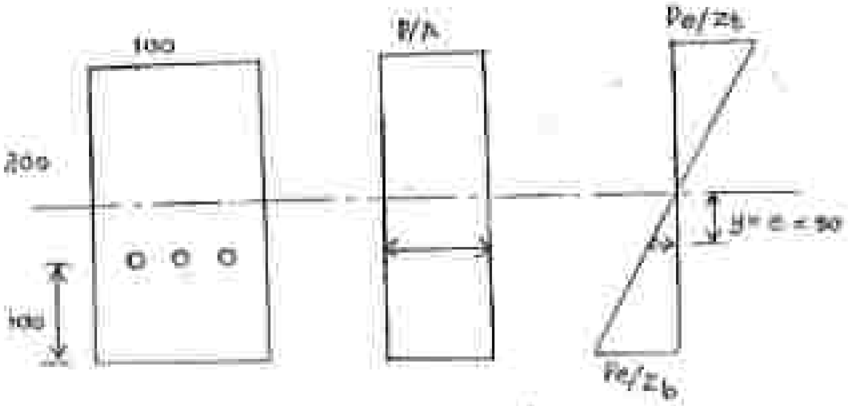
Wire tensioned	Loss in 1st	2nd	3rd	4th
1 wire	0	—	—	—
2 wire	mfc	0	0	0
3 wire	mfc	mfc	0	0
4 wire	mfc	mfc	mfc	0

Total loss of prestress in all wires $= 3\,mf_c + 2\,mf_c + mf_c$

$= 6\,mf_c$

$$\boxed{\left.\begin{array}{l}\text{Total loss of prestress}\\ \text{in all wires}\end{array}\right\} = \frac{n(n-1)}{2}\,(mf_c)}$$

P-83.
Q.5



Post tensioned case:

Prestress in steel, $\sigma_s = 1200$ MPa.

Prestressing force in each wire, $P = \sigma_s A_s = 1200 \times 50$

$= \underline{60 \text{ kN}}$

$$f_c = \frac{P}{A} + \frac{Pe}{I}(e)$$

gross c/s area

$$f_c = \frac{60\times10^3}{100\times300} + \frac{60\times10^3\times 50}{100\times\frac{300^3}{12}}\times 50 = \underline{\underline{2.67 \text{ MPa}}}$$

Simultaneous tensioning and anchoring of all three cables is done $\Rightarrow$ Loss of stress $= \underline{\underline{0}}$

6. Successive tensioning:

$$\text{Loss of prestress in steel} = \frac{n(n-1)}{2}\, m f_c$$

$$= \frac{3\times2}{2}\times 6\times 2.67$$

$$= \underline{\underline{48.06 \text{ MPa}}}$$

$$\% \text{ loss} = \frac{\text{loss}}{\text{initial stress}}\times100 = \frac{48.06}{1200}\times100$$

$$= \underline{\underline{4\%}}$$

04. $$\delta l = \frac{Pl}{A_s E_s}$$

$$20 = \frac{P\times 10{,}000}{500\times2\times10^5}$$

Prestressing force developed in wire, P = 200 kN.

$$m = \frac{E_s}{E_c} = 10$$

$$f_c = \frac{P}{A} + \cancel{\frac{Pe^2}{I}}^{\,0}$$

$$= \frac{200\times10^3}{200\times400} = 2.5 \text{ MPa}$$

Loss of prestress $= m f_c = 10\times2.5 = \underline{\underline{25 \text{ MPa}}}$

Prestress in steel, $\sigma_i = \frac{P}{A_s} = \frac{200\times10^3}{500} = 400$ MPa.

% loss $= \frac{\text{Loss}}{\text{Initial stress}} \times 100 = \frac{25}{400}\times100$

$= \underline{\underline{6.25\%}}$

→ Loss due to Shrinkage of Concrete.

– long term (gradual loss) loss as shrinkage due to evaporation of moisture takes minimum of one year.

– This loss occurs both in pretensioned and post tensioned members. However higher % of loss will be in pre tensioned members.

Loss of prestress $= \epsilon_{sc}\ E_s$

where $\underset{\text{shrinkage}}{\epsilon_{sc}} = \frac{dl}{l}$; shrinkage strain in concrete.

dl → deformation due to shrinkage.

• ϵ_{sc} is given by code as follows :-

(i) Pre tension → $\epsilon_{sc} = 0.0003 = 3\times10^{-4}$

(ii) Post tensioning → $\epsilon_{sc} = \frac{0.0002}{\log_{10}(t+2)}$

t → age of concrete (in days) at which prestressing is done

⊙ For pretensioned members

Loss due shrinkage of concrete $= \epsilon_{sc} E_s$

$= 0.0003 \times 2 \times 10^5$

NOTE:

$= 60$ MPa

⊙ Even for RCC, the strain due to shrinkage of concrete is 0.0003

⊙ For post tensioned members

– Age @ transfer of prestress → 3 months = 90 days.

Loss due to shrinkage of concrete $= \epsilon_{sc} E_s$

$$= \frac{0.0002}{\log_{10}(90+2)} \times 2 \times 10^5$$

$= \underline{20.4 \text{ MPa}}$

→ Loss due to Creep of Concrete

– Due to sustained or constant prestressing force steel wire undergoes loss.

– This is also a long term (gradual) loss.

– This occurs in both pretensioning & post tensioning.

* Ultimate Creep Strain Method.

Loss of prestress in steel $= \underset{\text{creep}}{\epsilon_{cc}} \, \underset{\text{concrete}}{E_s}$

where ϵ_{cc} → ultimate creep strain of concrete.

* Creep Coefficient Method.

Loss of prestress in steel $= \phi (m f_c)$

ϕ → creep coefficient.

ϕ values

7 days = 2.2

28 days = 1.6

1 year = 1.1

NOTE:

- The most critical loss among all is shrinkage of concrete, then creep of concrete.

→ Loss due to Creep of Steel.

- It is known as "Relaxation of prestress in steel"
- gradual loss

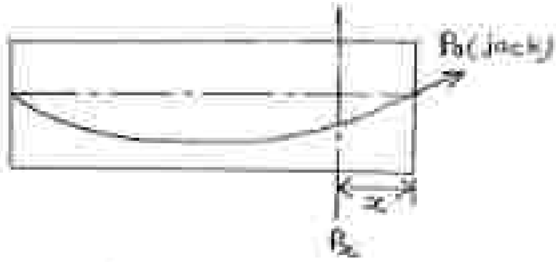
$$\text{Loss} = 2\% \text{ to } 8\% \text{ initial stress}$$

→ Friction loss

- only in post tensioned members
- immediate loss, at the time of tensioning only.

P_0 → initial prestressing force at jacking end.

P_x → prestressing force at a distance of x from jacking end.



$$P_x < P \text{ (due to loss)}$$

$$P_x = P_0 \, e^{-(\mu\alpha + kx)}$$

But $e^x = x^0 + \dfrac{x^1}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$

Neglect higher order powers (≥ 2) & use $x = -(\mu\alpha + kx)$.

$$P_x = P_0 \left(1 - (\mu\alpha + kx)\right)$$

$$P_x = P_0 - P_0 (\mu\alpha + kx)$$

$$\boxed{\text{Loss of prestress over a distance } x = P_0(\mu\alpha + kx).}$$

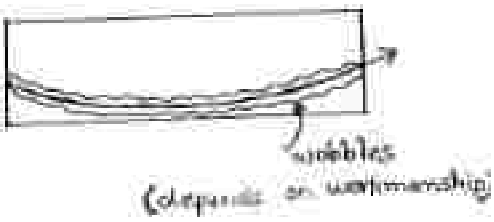
where $\mu \rightarrow$ coefficient of friction b/w cable & duct (0.25 – 0.55)

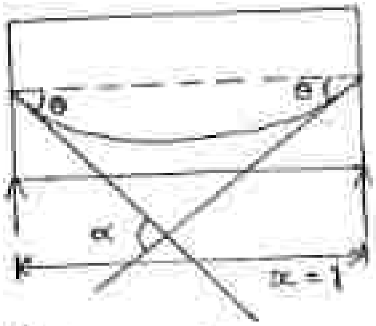
$K \rightarrow$ wobbling constant (…/m).

$\alpha \rightarrow$ cumulative angle in radians through which tangent to the cable profile has turned b/w any two points under consideration (= 0 for straight cables).

⊙ For straight cable, $(\alpha = 0)$

$$\boxed{\text{Loss} = P_0 kx}$$





→ Equation of parabola:

$$y = \frac{4hx}{l^2}(l-x)$$

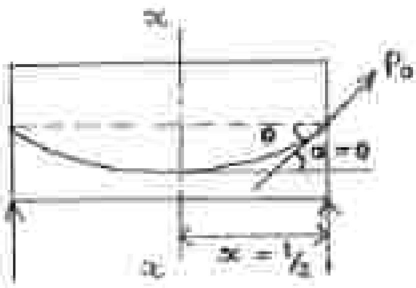
⊙ Jacking from one ends.

$$\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x).$$

At A, $x = 0 \Rightarrow \boxed{\theta = \frac{4h}{l}}$

$$\alpha = 2\theta = \frac{8h}{l}.$$

⊙ Jacking from two ends reduces friction loss by exactly 50%

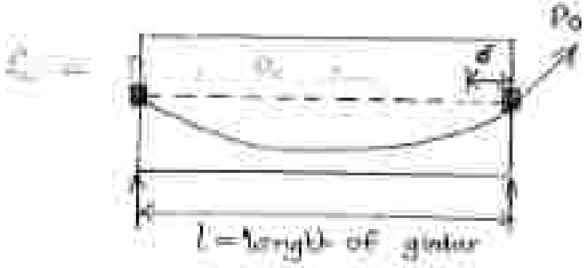


$$\theta = \alpha = \frac{4h}{l}$$

→ Loss due to Anchorage Slip.

- only in post tensioned.
- immediate loss occurs at the time of anchoring

$$\delta = \left(\frac{P}{A}\right)\left(\frac{l}{E}\right)$$



Loss of prestress $= \frac{\delta \cdot E_s}{l}$

where $\delta \to$ anchorage slip

* Total % loss

Pretensioning $= 18\%$

Post tensioning $= 15\%$

P-82

7. Initial stress $= 1200$ MPa

$\delta = 3$ mm

Loss of prestress $= \frac{\delta E_s}{l} = \frac{3 \times 2.1 \times 10^5}{30 \times 10^3} = \underline{\underline{21 \text{ MPa}}}$

% loss $= \frac{21}{1200} \times 100 = \underline{\underline{1.75\%}}$

9. Loss of prestress $= \epsilon E = 0.0008 \times 200 \times 10^3$

$= \underline{\underline{160 \text{ MPa}}}$

Stress remaining after loss $= 200 - 160 = \underline{\underline{40 \text{ MPa}}}$

8. $b = 120$ mm, $D = 200$ mm, $P = 150$ kN, $e = 20$ mm

$$m = \frac{E_s}{E_c} = \frac{2.1 \times 10^5}{3 \times 10^4} = \underline{\underline{7}}$$

$$f_c = \frac{P}{A} + \frac{Pe^2}{I} = \frac{150\times10^3}{120\times200} + \frac{150\times10^3\times20^2}{\frac{120\times200^3}{12}}$$

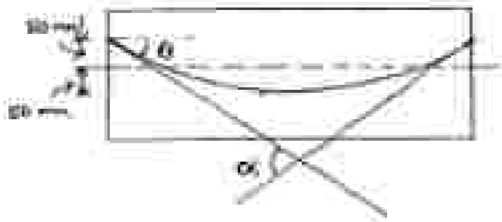
$$= 7 \text{ MPa.}$$

Loss of stress in steel $= m f_c = 7\times7 = \underline{49 \text{ MPa}}$

Initial prestress $= \frac{P}{A_{\text{steel wire}}} = \frac{150\times10^3}{187.5} = \underline{800} \text{ MPa}$

$\therefore$ % loss of stress $= \frac{49}{800}\times100 = \underline{\underline{6.125\%}}$

1. Tensioned from one end.



$\alpha = 2\theta = \frac{8h}{l}$

$= \frac{8\times(50+50)}{10000} = 0.08$ rad.

$\mu = 0.35$, $k = 0.0015$ per m.

Loss of stress $= P_0(\mu\alpha + kx)$

$= 1200(0.35\times0.08 + 0.0015\times10)$

$= 0.043 \times 1200 \text{ MPa} = \underline{51.6} \text{ MPa}$

% loss of stress $= \frac{0.043\times1200}{1200}\times100 = \underline{\underline{4.3\%}}$

2. Tensioned from both the ends.

$\alpha = \theta = \frac{4h}{l} = 0.04$ rad.

Loss of stress $= P_0(\mu\alpha + kx)$

$= 1200(0.35\times0.04 + 0.0015\times5) = 25.8$

% loss $= \frac{25.8}{1200} = \underline{\underline{2.15\%}}$

3. Cable is straight & tensioned from one end $\Rightarrow$ loss $= P_0 kx = 1200\times0.0015\times10 = 18$

% loss $= \frac{18}{1200}\times100 = \underline{1.5\%}$

16th nov,
WEDNESDAY

16. CEMENT

- Cement is invented by Joseph Aspdin (1824)
- Cement is the binding material in concrete, whereas fine aggregate is the void filler and coarse aggregates imparts strength.
- Cement is manufactured at a temperature of 1300°c to 1500°.
- Gypsum is added to (2 to 3%) prevent flash setting
- Methods of manufacture : (i) Dry process
  (ii) Wet process.

→ Chemical Composition

1. CaO — controls strength & soundness
2. SiO_2 — gives strength, excess causes slow setting
3. Al_2O_3 — for quick setting, excess lowers strength.
4. Fe_2O_3 — responsible for colour.
5. MgO — colour and hardness
6. Alkalies — causes efflorescence & cracking

→ Composition of Cement Clinker

(i) Tricalcium Silicate (Alite) — C_3S
- Early strength (ie 7 day strength)
- 7 day strength = $\frac{2}{3}$ (28 day strength)
- more heat of hydration. (120 cal/g).

(ii) Dicalcium Silicate (Blite) — C_2S
- later strength (after 7 day).
- less heat of hydration (60 cal/g)

(iii) Tricalcium Aluminate (Clite) – C_3A

- Very high heat of hydration (320 cal/g)
- initial strength
- avoid usage in coastal areas.

(iv) Tetra Calcium Alumino Ferrite (Flite) – C_4AF

- No strength.
- Very high heat of hydration → more cracking

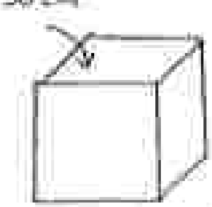
→ Types of Cements.

(i) Ordinary Portland Cement – OPC.

At 28 days, 80% of strength is attained by cement and it can take full design loads from 28th day. Based on 28 day compressive strength of cement mortar, different grades of cement are:

33 G	43 G	53 G
(outdated)		(most common)

* Compressive strength test:



cement mortar cube

- Based on IS:516 recommendations.
- Surface area of one face = 50 cm²
- Size of one side = $\sqrt{50}$ = 7.07 cm
- Mortars are prepared with

  Cement : Sand = 1:3

  Ennore sand (uniform sand) is the preferred

- Water = $\left(\frac{P}{4}+3\right)$(wt. of sand + cement)

where P → Normal or standard consistency of cement (For OPC, P ≃ 30%)

- 3 cubes are prepared and immersed in water and tested for compressive strength after 28 days
- Variation in strength ≯ 15%

$\boxed{\uparrow \text{Fineness} \Rightarrow \uparrow \text{strength.}}$

(ii) Portland Pozzolana Cement – PPC

- 33 G (outdated)   43 G   53 G

- Pozzolana : siliceous material which has no cementitious properties when it is used alone but in the presence of cement it posess cementitious properties
- Natural Pozzolanas :
  (i) Burnt Clay
  (ii) Pumicite
  (iii) Diatomaceous earth.
- Artificial Pozzolanas :
  (i) Fly ash.
  (ii) Silica Fume
  (iii) GGBS (Ground Granulated Blast Furnace Slag)

Flyash is a pozzolana obtained as a by-product in Thermal power plants 30% flyash is added to grinded portland cement clinker. Although the strength is gained slowly, 90 day strength is more than that of OPC. So OPC is now replaced with PPC in markets.

- Read through all the other types of cement given in booklet.

→ Tests on Cement.

1. Fineness

– index of grinding.

– determined by sieving through 90 μ sieve. Residue should not exceed 10% by weight for OPC.

– Blain's Air Permeability test : gives specific surface – the surface area of 1g of cement particles (cm^2/g)

2. Standard Consistency

– % water required to make workable cement paste.

– Vicat's Apparatus with plunger (1cm ϕ).

– For OPC, 30%.

3. Initial Setting Time.

– Time at which cement starts setting process.

– Vicat's Apparatus using Vicat's Needle (1mm square needle)

– For OPC, initial setting time ≮ 30 min.

4. Final Setting Time.

– Time at which cement ends setting process and becomes hard.

– Vicat's Apparatus using Vicat's Needle with annular collar of 5mm ϕ

– For OPC, final setting time ≯ 10 hours

5. Soundness Test.

– Expansion of cement due to presence of free lime and magnesia is called Unsoundness

– Determined by Le-chatlier Apparatus.

– Auto clave test : quick test.

→ Heat of Hydration.

- Due to addition of water, exothermic chemical reactions occur.
- Heat evolved from concrete causes cracks.
- Heat of hydration determined by: Adiabatic Calorimeter test or Vacuum Flask test.

→ Specific Gravity

- using Kerosene and sp.gr. bottle at 27°C.
- For OPC, specific gravity is around 3.1

12th nov,
WEDNESDAY

17 AGGREGATES

Sand :

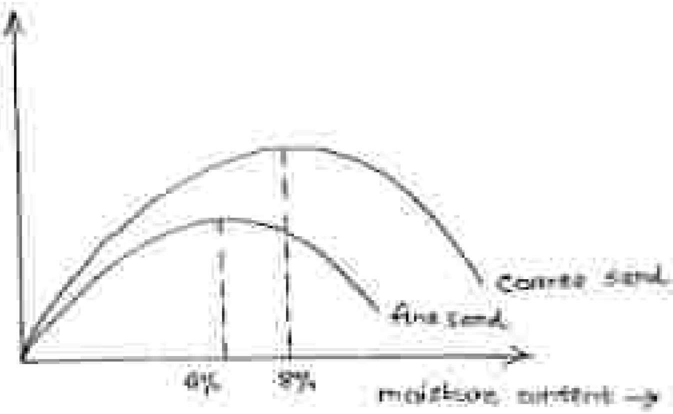
- Well graded (river sand)
- Robo sand (artificial)

Gravel :

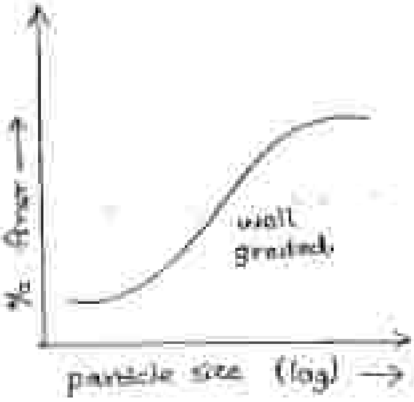
- Hard blasted granite chips

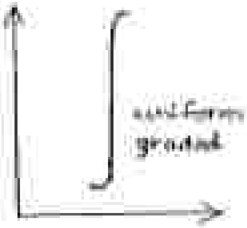
→ Bulking of Sand

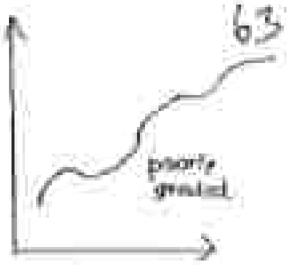
- Increase in volume due to adhered moisture
- negligible in case of coarse aggregates
- ↑ fineness ⇒ ↑ bulking
- fine sand shows max. bulking at 4% – 6% moisture, whereas coarse sand shows at 8%



→ Grading of Aggregates







- sieve sizes 80mm to 150 μ are used in sieve analysis

→ Fineness Modulus

$$FM = \frac{\text{Cumulative \% material retained}}{100}$$

- Recommended FM of coarse aggregate is 7

-

Fine sand	2.2 – 2.6
Medium sand.	2.6 – 2.9
Coarse sand	2.9 – 3.2

- sands with FM >2 should not be used in construction

- For RCC, medium sand of zone II as per BIS should be used.

12th nov
wednesday

18 CONCRETE

→ Concrete Mixes

* Nominal mix
  - based on volume proportion
  - upto M20 can be designed by this method.

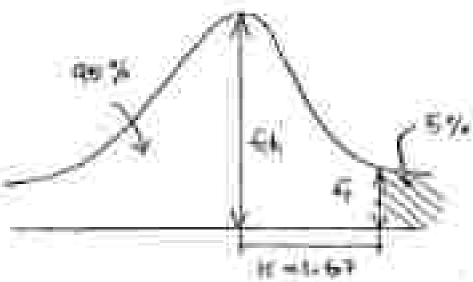
Grade.	C	:	FA	:	CA
M15	1	:	2	:	4
M20	1	:	1.5	:	3

  - bulking should be taken care of (volume).

* Standard mix
  - based on dry weights of ingredients
* Design mix
  - Scientific method of design based on IS : 10262
  - weight proportions
  - No fixed proportions for a specified strength.

$$\boxed{\text{Target strength, } f_t = f_{ck} + K(s)}$$

S → standard deviation (given in code based on qty control)

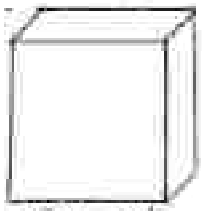


∴ In Limit State method
- Design load, F = $F_m + K(s)$
- Characteristic strength, $f = f_m - K(s)$.

→ Properties of Hardened Concrete

1. Compressive Strength

- determined based on cube test.
- for random strength : 3 cubes avg. Variation ≯ ±15%
- for characteristic compressive strength : 30 samples. Variation > 15% is also allowed.
- Grade of concrete is based on standard (15 cm) cube at 28 day strength.



15 cm cube.

⊙ 10 cm cube (IS: 516)

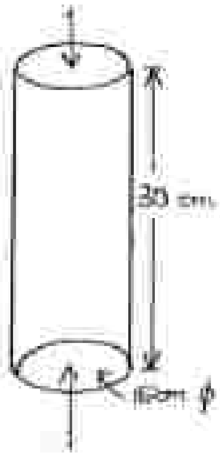
$\sigma_{10\,cm} = 10\% \uparrow \sigma_{15cm}$

$\boxed{\sigma_{10\,cm} = 1.1\ \sigma_{15\,cm}}$ (more volume ⇒ less strength)

$\sigma_{\text{concrete in a structure}} = 0.67\ f_{ck}$ ← grade of conc. based on 15 cm cube

↳ 33% ↓ in strength based on ↑ λ

⊙ Cylinder Compressive strength (IS: 516)



$\lambda_{cylinder} = \frac{l}{b} = \frac{30}{15} = 2$

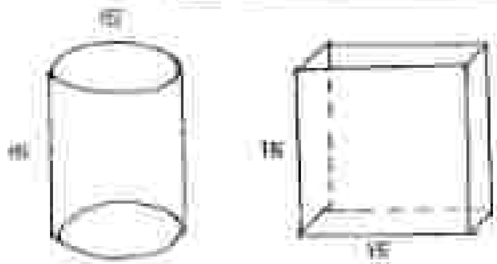
$\lambda_{cube} = \frac{l}{b} = \frac{15}{15} = 1.$

↑λ compared to a standard cube (↓ strength)

$\sigma_{cylinder} = 0.8\ (\sigma_{15cm\ cube})$

$\boxed{\sigma_{15cm\ cube} = 1.25\ (\sigma_{cylinder})}$

Q



λ = 1 (for both cube & cylinder)

$Vol_{(cube)} > Vol_{(cylinder)}$

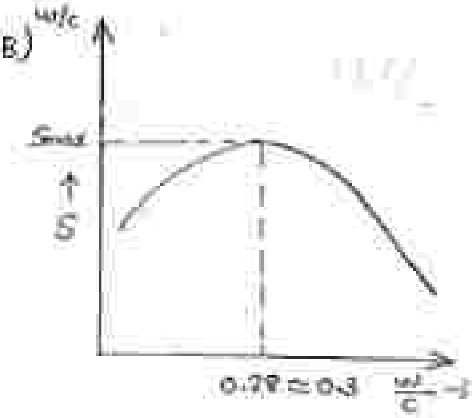
$Strength_{(cube)} < Strength_{(cylinder)}$

→ Properties of Fresh Concrete

1. Workability

– relative ease with which concrete can be mixed, transpo moulded and compacted.

* Abraham Equation : $S = (A/B)^{w/c}$



where $S \rightarrow$ compressive strength (MPa).

$w/c \rightarrow$ water cement ratio

* Workability Test

(i) Slump Test

– field test

– based on height of fall

– trench fill, insitu piles, tremie conc : 100 (min)

↑ ht. of fall ⇒ ↑ workability. 150 (max.)

(ii) Compaction Factor Test

⊛ ↑ compaction factor ⇒ ↑ workability.

Compaction factor $= \dfrac{\text{wt of partially compacted concrete}}{\text{wt of fully compacted concrete}}$

(iii) Vee-bee Consistometer Test.

– lab test

⊛ ↑ workability ⇒ ↓ vee bee time (in s).

(iv) Flow Table Test

⊛ ↑ % flow ⇒ ↑ workability

(v) Kelley Ball test

– used for finished concrete

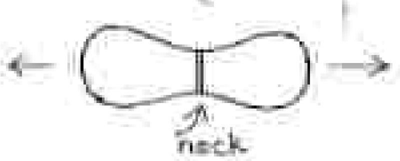
⊛ ↑ depth of penetration ⇒ ↑ workability

2. Tensile strength of concrete.

(i) Direct Tensile Strength

Eg: Anchor piles, side walls of circular water tanks

– compound lever apparatus

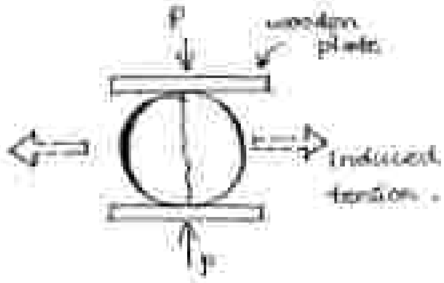


$\sigma_{DT} = \dfrac{\text{load @ failure}}{\text{c/s area @ neck}}$

$\sigma_{\substack{\text{direct}\\\text{tension}}} = 10 \text{ to } 15\% \ \sigma_{\substack{\text{comp. strength}\\(f_{ck})}}$

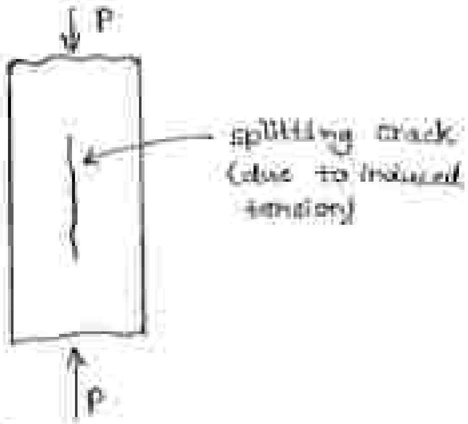
(ii) Indirect Tension.

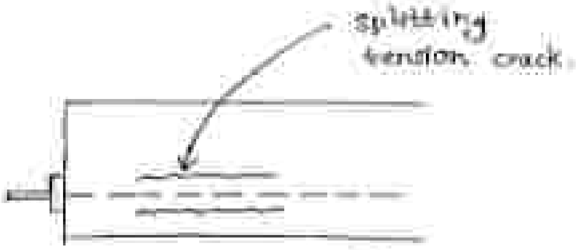
a) Split Cylinder Test
(Brazilian test)



$$\boxed{\sigma_{split} = \frac{2P}{\pi D L}}$$

where D → diameter (15 cm)
L → length (30 cm)





▪ PSC beam and block.

▪ column

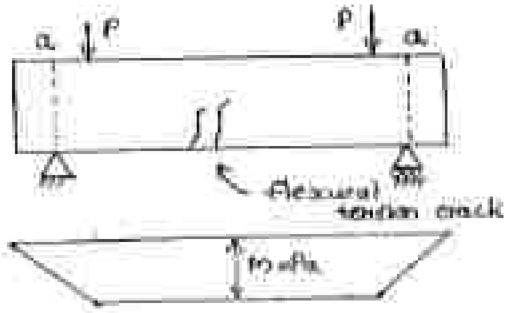
b) Modulus of Rupture (fcr)

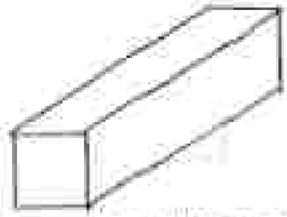
– common in any flexural member, beams etc

– tensile strength of concrete in bending tension.

- flexural tensile strength.

* Prism test (PCC beam).





15 cm x 15cm x 40 cm (std. size).

Use flexural equation,

$$\frac{M}{I} = \frac{f_{cr}}{y}$$

$$f_{cr} = \frac{M}{Z}.$$

For standard prism with two point loadings (pure bending).

- $M = Pa$

  $P \rightarrow$ load at cracking

  $Z = \frac{15 \times 15^2}{6}$

As per IS 456, emperical formula

$$f_{cr} = 0.7\sqrt{f_{ck}} \quad \text{(based on Prism test).}$$

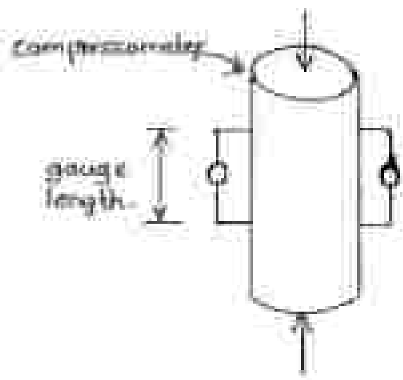
$$\sigma_{split} = \frac{2}{3} f_{cr}$$

3. Modulus of Elasticity of Concrete.

- standard cylinder test (axial compression)

$$\text{Gauge length} = 5.65\sqrt{A}$$

where $A \rightarrow$ initial or nominal c/s area.



(i) Initial tangent modulus (E_{it})

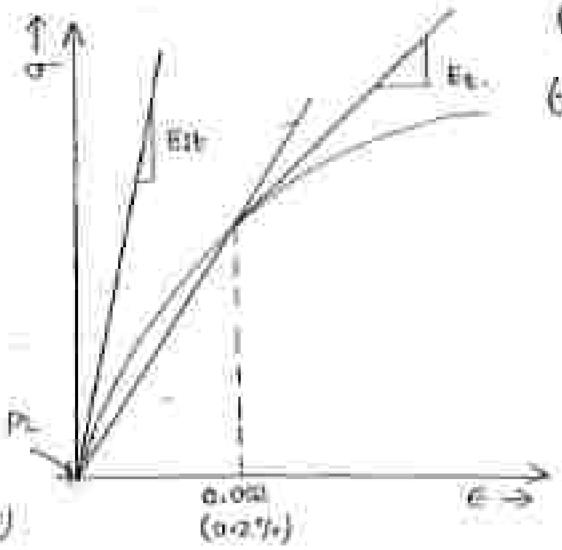
(ii) Tangent modulus (E_t)

(iii) Secant modulus.

$$\boxed{E_c = 5000\sqrt{f_{ck}}}$$

based on Secant method.

$E_c \rightarrow$ short term modulus of concrete.



→ Non Destructive Testing (NDT)

– testing methods of concrete members in a structure without disturbing its state.