COURSE MATERIAL

DIPLOMA IN 6^{TH} SEMESTER MECHANICAL ENGINEERING

LECTURE NOTES

ON
INDUSTRIAL ENGINEERING AND MANAGEMENT

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INDUSTRIAL ENGINEERING AND MANAGEMENT (TH-1)
CHAPTER- : PRODUCTION PLANNING &CONTROL (PPC)

1. Definition & objective of PPC

Introduction:

Production is an organized activity of converting raw materials into useful products. But before starting the actual production process planning is done –

- To anticipate possible difficulties.
- To decide in advance – how the production processes be carried out in a best & economical way to satisfy customers.

However, only planning of production is not sufficient. Hence management must take all possible steps to see that plans chalked out by the planning department are properly adhered to and the standard sets are attained. In order to achieve it, control over production process is exercised.

Objective:

Therefore, the ultimate objective of production planning and control is to produce products of

- right quality
- in right quantity
- at right time

By using the best and least expensive methods/procedure.

Definition:

PPC may be defined as the direction and co-ordination of the firms materials and physical facilities towards the attainment of pre-specified production goals in the most efficient and economical way.

Function of PPC:

The various functions of PPC dept. can be systematically written as:

1. Planning phase
   - Prior planning
     - forecasting
     - Order writing
     - Product design information
   - Active planning
     - Process planning & routing
     - Materials req. planning
     - tools req. planning
     - Loading
     - scheduling

Action phase – Dispatching

Control phase
- Progress reporting
- Corrective action

Explanation of each term

(a) Forecasting: Estimation of quality & quantity of future work.

(b) Order writing: Giving authority to one or more persons to do a particular job.

(c) Product design information: Collection of information regarding specification, bill of materials, drawing.

(d) Process planning and routing: Finding the most economical process of doing work and then deciding how and where the work will be done?

(e) Materials planning: It involves the determination of materials requirement.

(f) Tools planning: It involves the requirements of tools to be used.

(g) Loading: Assignment of work to men & m/c.

(h) Scheduling: When and in what sequence the work will be carried out. It fixes the starting and finishing time for the job.

(i) Dispatching: It is the transition from planning to action phase. In this phase the worker is ordered to start the actual work.

(j) Progress reporting:

i. Data regarding the job progress in collected.

ii. It is compared with the present level of performance.

(k) Corrective action: Expediting the action if the progress deviates from the planning.

(c) Aggregate Planning

Intermediate range planning which is done for a period of 3-12 months of duration is called Aggregate Planning as obvious from the following diagram.

Planning process

Long range planning (= strategic planning)(for 1-5 years of duration)	Intermediate range planning (=aggregate planning)(for 3-12 months)	Short term planning (for scheduling and planning for day to day shop floor activities). (for 1-90 days)

Aggregate plans acts as an interface (as shown below by planning hierarchy) between strategic decision and short term planning.

Planning hierarchy

Strategic decision

(1-5 months)

↓

Aggregate planning

(3-12 months)

↓

Short term planning

(1-90 days)

Aggregate planning typically focuses on manipulating several aspects of operations –

- Aggregate production volume
- Inventory level
- Personal level
- Machinery & other facility level

To minimize the total cost over some planning horizon while satisfying demand and policy requirements.

In brief, the objectives of aggregate planning are to develop plans that are feasible and optimal.

Aggregate Planning

Aggregate production planning (it indicates the level of output)	Aggregate capacity planning (keep desired level)

Characteristic of aggregates planning

Forecasting:

The aggregate plan is based on satisfying expected intermediate- term demand, so accurate forecasts of these demands are necessary, because seasonal variation patterns are usually important in aggregate planning.

In addition to demand, wage rates, material prices and holding costs can change enough to affect the optimal plans. But these forecasts are relatively easy to obtain because, they are specified in contractual agreements.

Identifying the planning variables

The two most important planning variables are:

- The amount of products to produce during each time period. and
- The amount of direct labours needed.

Two in-direct variables are:

- The amount of product to add to/remove from inventory.
- The amount of workforce/labour should be increased/decreased.

Implementing an Aggregate plan

Aggregate plans are normally generated by using → Optimisation method.

During a planning period

- Employees may produce more/less than expected.
- Actual demand may not be same as predicted.
- More employees leave the company than expected.
- More/less may be hired than expected
- Inventory may sometimes be damaged and so on.

Therefore, the 6-12 months aggregate plan devised for one period may no longer be optimal for the next several months.

We do not simply generate one plan for the next 12months and keep that plan. Until it has been completely implemented. Aggregate planning is a dynamic process that requires constant updating.

In actual practice, we first develop an aggregate plan that identifies the best things to do during each period of planning horizon to optimize the long term goal of the organization. We then implement only the 1st period of plan; as more information becomes available, we update and revise the plan. Then action is implemented in the first period of the revised plan, gather more information and update again. This is illustrated in the following.

Implement 1st period	Jan	Feb	Mar	April	May	June

↓ Update & revise the plan

Implement 1st period	Feb	Mar	April	May	June	July

↓ Update and revise the plan

Implement 1st period	Mar	April	May	June	July	Aug ↓

Decision option in Aggregate Planning

The decision options are basically of 2 types.

i. Modification of demand
ii. Modification of supply.

i. Modification of demand

The demand can be modified in several ways-

a) **Differential pricing**

It is often used to reduce the peak demand or to increase the off period demand. Some examples are:

- Reducing the rates of off season fan/woolen items.

- Reducing the hotel rates in off season.
- Reducing the electric charge in late night etc.

b) **Advertising and promotion**

These methods are used to stimulate/smooth out the demand. The time for the advertisement is so regulated as to increase the demand during off period and to shift demand from peal period to the off period.

c) **Backlogs**

Through the creation of backlogs, the manufacturer ask customer to wait for the delivery of the product, thereby shifting te demand from peak period to off period.

d) **Development of complementary products**

Manufacturer who produce products which are highly seasonal in nature, apply this technique. Ex- Refrigerator Company produce room heater. TV Company produce DVD etc.

<u>ii.</u> Modification of supply

There are various methods of modification of supply

a) **Hiring ad lay off employees**

The policy varies from company to company. The men power/work force varies from peak period to slack/of period. Accordingly hiring/lay off employee is followed without affecting the employee morale.

b) **Overtime and under time**

Overtime and under time are common option used in cases of temporary change of demand.

c) **Use of part time or temporary labour**

This method is attractive as payment for part time/temporary labour is less.

d) **Subcontracting**

The subcontracting may supply the entire product/some of the components needed for the products.

e) **Carrying inventories**

It is used by manufacturer who produces item in a particular season and sell them throughout the year.

Aggregate planning strategies

Pure strategy	Mixed strategy
If the demand and supply is regulated by any one of the following strategies. (a) Utilizing inventory through constant workforce. (b) Varying the size of workforce. (c) Sub contracting (d) Making changes in demand pattern.	If the demand and supply is regulated by mixture of the strategies as mentioned aside, it is called mixed strategy.

(C) Materials Requirement Planning (MRP)

In manufacturing a product, the firm has to plan materials so that right quantity of materials is available at the right time for each component/subassembly of the product. The various activities interlinked with MRP is stated in the following.

Objective of MRP

1. **Inventory Reduction**: MRP determines how many of a component are needed and when to meet the master production schedule. It enables the manager to procure that components as and when it is needed. As a result it avoids cost of carrying inventory.
2. **Reduction in production and Delivery Lead Time:**
   MRP co-ordinates inventories procurement and production decision and it helps in delay in production.
3. **Realistic commitment**: By using MRP in production the likely delivery time to customers can be given.

4. **Increased Efficiency**: MRP provides close co-ordination among various departments and work centers as product buildup progresses through them. Consequently, the production can proceed with fewer indirect personnel.

MRP calculation

The terminologies which are involved in doing MRP calculations are:

- Projected requirements
- Planned order release
- Economic order quantity
- Scheduled receipts (receipts)
- Stock on hand

Master production schedule gives particulars about demands of the final assembly for the period in the planning horizon. These are known a projected requirements of the final assembly.

The projected requirements of the subassemblies which are in the next immediate level just below the final product can be calculated only after completing MRP calculation for the final products. Similarly the projected requirements of the subassemblies which are in the 2nd level can be calculated only after completing the MRP calculation for the respective subassemblies in the 1st level. Like this the projected requirements for all subassemblies can be calculated.

Stock on hand is the level of inventory at the end of each period. Generally the initial on hand quality if exists for the final product/each subassembly is given in the input. For each period, the stock on hand is computed by using the following formula.

$SOH_t = SOH_{t-1} + R_t - PR_t$ ------------------------(1)

Where, SOH_t = Stock on hand at the end of period t.

SOH_{t-1} = Stock on hand at the end of period t-1.

R_t = The scheduled receipt at the beginning of the period t through an order which has been placed at some early period.

PR_t = Projected requirement of the period t.

Planned order release is the plan (i.e., quantity and date) to initiate the purchase. The planned order release for the period t is nothing but placing an order if the stock on hand (SOH_t) at the beginning of period t is less than the projected requirement (PR_t). Genarally the size of the order = Economic Order Quantity (EOQ).

The EOQ is calculated by using the following formula

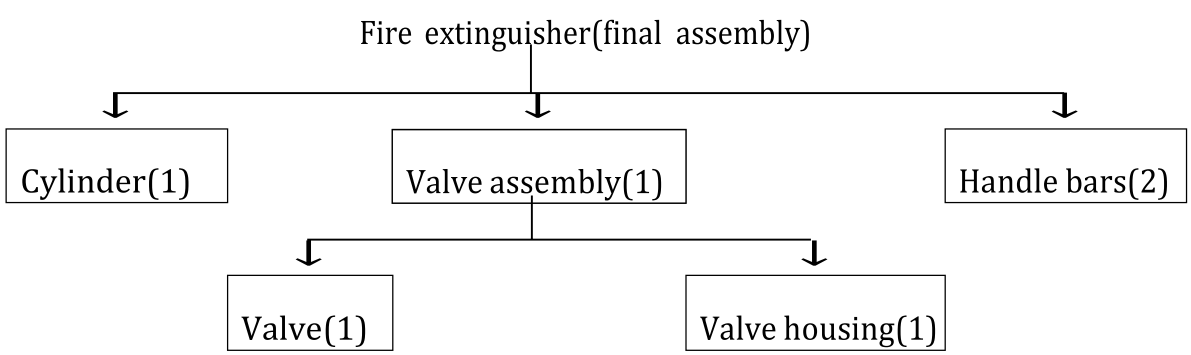
i.e., $EOQ = \sqrt{\frac{2CoD}{Ci}}$

where, D = Average demand/week

Co = ordering cost, Ci = earning same time cost/week

Example (to demonstrate MRP calculation)

In order to demonstrate the working of MRP, let us consider manufacturing of five extinguisher as stated in the following.



The master production schedule to manufacture the fire extinguisher is given in Tab-1.

Tab-1: Master production schedule

Week	1	2	3	4	5	6	7	8
Demand	100		150	140	200	140		300

The details of bill of materials along with economic order quantity and stock on for the final product and subassemblies are shown in Tab-2.

Tab-2: Details of Bill of materials

Parts required	Order quantity	No. of units	Lead time (week)	Stock on hand
Fire extinguisher	300	1	1	150
Cylinder	450	1	2	350
Valve assemblies	400	1	1	325
Valve	350	1	1	150
Valve housing	450	1	1	350
Handle bars	700	2	1	650

Lead time internal between placement of order receipt of materials.

Complete the material requirements plan for the fire extinguisher, cylinder, valve assembly, valve, valve housing, and handle bars and show what they must be released in order to satisfy the master production schedule.

Solution:

(a) MRP calculation for fire extinguisher

The projected requirements for the fire extinguisher is same as master production schedule as shown Tab-1.

One unit of fire extinguisher require

- One unit of cylinder

- One unit of valve assembly, and
- Two units of handle bars.

The MRP calculations for the fire extinguisher are shown in Tab-3.

Tab-3: MRP calculations for fire extinguisher

EOQ = 300, Lead time = 1 week

Period	0	1	2	3	4	5	6	7	8
Projected requirement		100		150	140	200	140		300
Stock in hand	150	50	50	200 (-100)	60	160 (-140)	20	20	20 (-280)
Planned order release			300		300			300	
Receipt				300		300			300

Similarly the MRP calculation may also be carried out for other components.

Routing

Routing may be defined as the "selection of proper follow which each part of the product will follow, while being transferred from raw material to finished products. Path of the products will also give sequence of operations to be adopted while manufacturing."

In other words, routing means determination of most advantageous path to be followed from department to department and machine to machine till raw materials get its final shape.

Routing determines the best and cheapest sequence of operations and to see that this sequence is rigidly followed.

Routing is an important function of PPC because it has a direct bearing on the "time" as well as "cost" of the operation. Defective routing may involve back tracking and long routes. This will unnecessarily prolong the processing time. moreover, it will increase the cost of material handling. Routing is affected by plant layout. In fact, routing and affected by plant layout are closely related. In product layout the routing is short and simple while under the process layout it tends to be long and complex.

Routing Procedure

1. **Analysis of the product:** the finished product is analysed and broken into number of components required for the product.
2. **Make and buy decision:** It means to decide whether all components are to be manufactured in the plant or some are to be purchased from outside. make and buy decision depends upon
    - The work load in the plant already existing
    - Availability of equipments
    - Availability of labour
    - Economy consideration
3. **Raw materials requirements**

    A part list and bill of materials is prepared showing name of part, quantity materials specification, amount of materials required etc.

4. Sequence of operations which the raw materials are to undergo are listed.
5. Machines to be used, their capacity is also listed.
6. Time required for each operation and subassemblies are listed.
7. The low size is also recorded.

The data thus obtained is utilized for preparing master route sheets and operation charts. the master route sheets give the information regarding the time when different activities are to be initiated and finished, to obtain the product and required time.

The next step is to prepare the route sheet for the individual item or component.

Route sheet

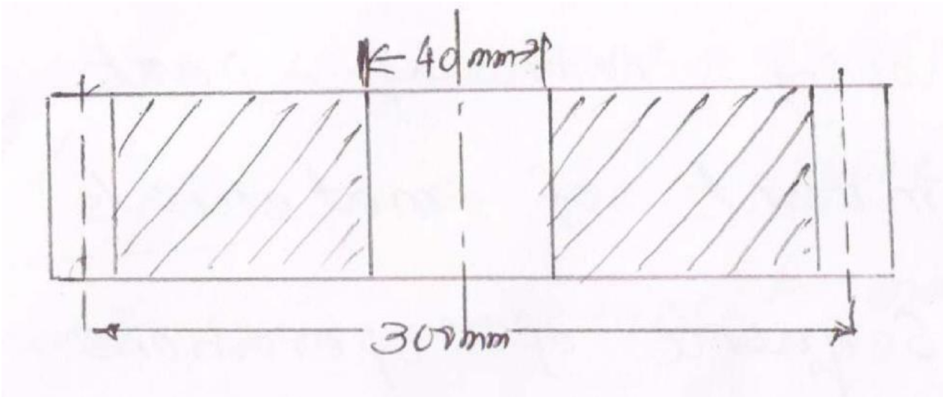
The operation sheet and the route sheet differs only slightly. An operation sheet shows everything about the operation i.e. operation descriptions, their sequence, type of machinery, tools, jigs & fixture required, setup & operation time etc. whereas, the route sheet also details the section (or department) and the particular machine on which the work is to be done. the operation sheet will remain the same if the order is repeated but the route sheet may have to be revised if certain machines are already engaged to order. except thin small difference, both seets contain practically the same iformation and thus generally combined into one sheet known as operation and route sheet as shown in fig 1.1.

Part no. – A/50

Name – Gear

Material – m.s.

Quantity – 100 Nos



Department	Machine	Operation	Description	Tool	Jigs/Fixture	time	
						setup	operation
Smithy	Power hammer PH/15	1 2	Forging Punching hole	- -	- -	4hrs 1hrs	30 min. 25 min.
Heat treatment	Furnace F/H/4	3	Normalizing	-	-	4 hrs	4 hrs.
Machine shop	Centre lathe CL-5	4	Face 2 end. Turn onter & inner face	Lathe tool	Chuck	15 min	1hr.
	Milling m/c Mm/15	5	Cut teeth	Side & face cutter	Dividing head	40 min	5 hrs
	Slotter SL/7	6	Make the key way	Slotting tool	-	10 min	30 min.

Advantages of Routing

1. Efficient use of available resources.

2. Reduction in manufacturing cost.
3. Improvement in quantity and quality of the o/p.
4. Provides the basis for scheduling and loading.

Scheduling

Scheduling may be defined as the assignment of work to the facility with the specification of time, and the sequence in which the work is to be done. Ex-time Table scheduling is actually time phasing of loading. the facility may be man power, machines or both. scheduling deals with orders and machines. it determines which order will be taken up on which machine in which department at what time and by which operator.

Objectives Loading and Scheduling

1. Scheduling aims to achieve the required rate of o/p with a minimum delay and disruption in processing.
2. To provide adequate quarters of goods necessary to maintain finished product at levels predetermined to meet the delivery commitment.
3. The aim of loading and scheduling is to have maximum utilization of men, machines and materials by maintaining a free flow of materials along the production line.
4. To prevent unbalanced allocation of time among production departments.
5. To keep the production cost minimum.

Factors Affecting Scheduling

(A) External Factors

1. Customers demand
2. Customers delivery dates
3. Stock of goods already lying with the dealers & retailers.

(B) Internal Factors

1. Stock of finished good with firm
2. Time interval to manufacture each component, subassembly and then assembly.
3. Availability of equipments & machinery their capacity & specification.

4. Availability of materials
5. Availability of manpower

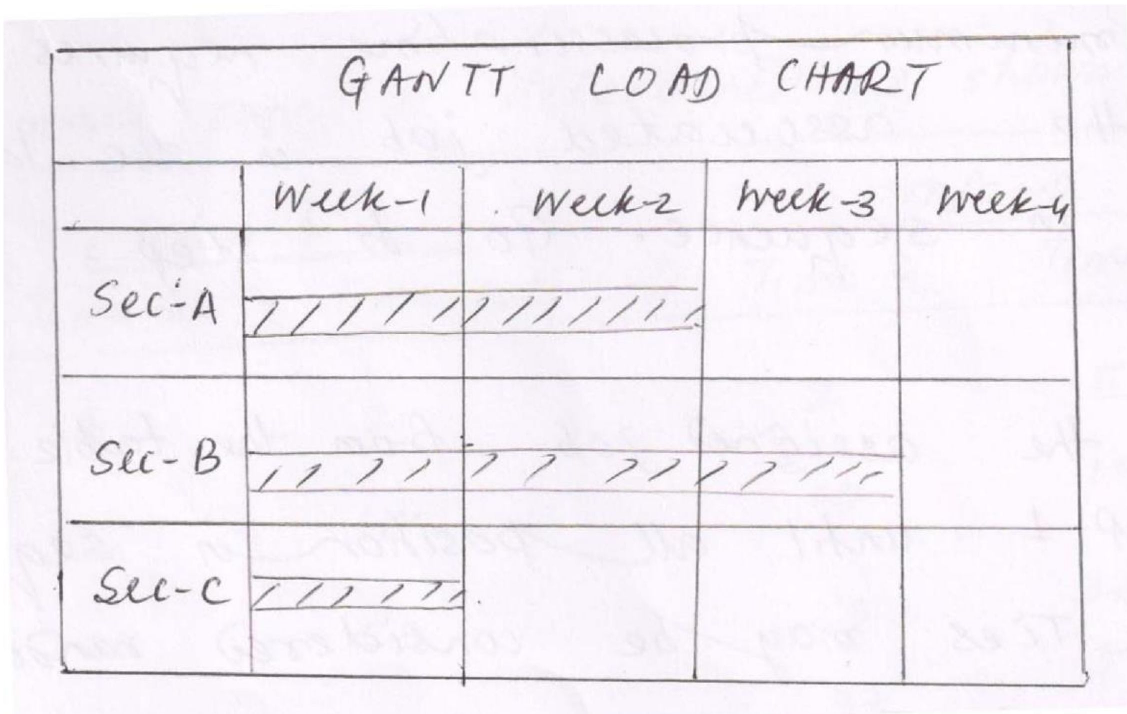
Scheduling Procedure

Scheduling normally starts with mater schedule. The following table shows master schedule for a foundry shop.

MASTER SCHEDULING FOR FOUNDRY SHOP. Maximum production capability/week = 100					
Order no.	Week-1	Week-2	Week-3	Week-4	Week-5
1.	15	18	20	15	18
2.	25	25	20	25	20

After master production schedule is made, the detailed schedules are thought of and made for each component, subassemblies, assemblies. The Gantt chars is a popular method commonly used in scheduling technique.

An example of Gantt chart is shown below. The hatched zone indicates actual work load against each section.



Instead of section, it may be m/c / other facilities. now a days computers are used to do this chart for different components/ m/c etc through readily available production software.

Machine loading using johnson's Rule

Loading may be defined as the assignment of work to a facility. the facility may be people, equipment, machine work groups or an entire plant. Therefore, machine loading is the process of assigning work to machine.

Johnson's Rule is most popular method of assigning jobs in a most optimum way such that the job can be produced with a minimum time & minimum idle time of the machine.

Case @ n Jobs in 2 machines

Job (s)	Machine-1	Machine-2
1	t_{11}	t_{12}
2	t_{21}	t_{22}
3	t_{31}	t_{32}
:	:	:
i	ti_1	ti_2
:	:	:
n	tn_1	tn_2

Methodology/Procedure

Step – 1. Find the minimum time among t_{i1} & t_{i2}

Step – 2 If the minimum processing time requires m/c-1, place the associated job in the 1st available position in sequence.

Step – 2 If the minimum processing time requires m/c-2 place the associated job in the last available position in sequence. Go to Step – 3.

Step – 3. Remove the assigned job from the table and return to step – 1 until all position in sequence are filled. (Tiles may be considered read only)

The above algorithm is illustrated with following example.

Example.- Consider the 2 machines and Six jobs follow shop scheduling problem. Using John son's algorithm obtain the optimal sequence which will minimize the make span. Find the value of make span.

Job	Time taken by m/cs, hr	
	1	2
1.	5	4
2.	2	3
3.	13	14
4.	10	1
5.	8	9
6.	12	11

Solution - The working of the algorithm is Summerised in the form of a table which is shown below.

Stage	Unscheduled	Min, tik	Assignment	Partial/full sequence
1. →	123456	t_{42}	Job 4 → [6]	*****4
2. →	12356	t_{21} →	Job 2 → [1]	2****4
3. →	1356 →	t_{12} →	Job1 → [5]	2***14
4. →	356 →	t_{51} →	Job 5 → [2]	25**14
5. →	36 →	t_{62} →	Job 6 → [4]	25*614
6. →	3 →	t_{31} →	Job 3 → [3]	253614

Now the optimum sequence – <u>2-5-3-6-1-4.</u>

The make span is determined as shown below

Job	m/c – 1		m/c – 2		Idle time on m/c - 2
	Time in	Time out	Time in	Time out	
2	0	2	2	5 →	2
5	2	10	10	19 →	5
3	10	23	23	37 →	4
6	23	35	37	48 →	0
1	35	40	48	52 →	0
4	40	50	52	53 →	0

The make span for this optimum schedule/assignment is 53 hrs.

Case (b) n jobs in 3 machines as shown is the following

Job	m/c-1	m/c-2	m/c-3
1	t_{11}	t_{12}	t_{13}
2	t_{21}	t_{22}	t_{23}
3	t_{31}	t_{32}	t_{33}
:	:	:	:
:	:	:	:
:	:	:	:
n	t_{n1}	t_{n2}	t_{n3}

One can extend the Johnson's algorithm to this problem if any of the following 2 conditions is satisfied.

If min of $t_{i1} \geq$ max t_{i2}

If min $t_{i3} \geq$ max t_{i2}, then an hypothetical problem with 2 machines and n jobs (as shown below) can be treated. the objective is to obtain optimal sequence for the data given in the following table. later the make span can be obtained (for the optimal sequence) using the data of original table.

Hypothetical 2 m/cs Problem

Job	Hyp m/c-A	Hyp m/c-B
1	$t_{11}+t_{12}$	$t_{12}+t_{13}$
2	$t_{21}+t_{22}$	$t_{22}+t_{23}$
:	:	:
i	$t_{i1}+t_{i2}$	$t_{i2}+t_{i3}$
:	:	:
n	$t_{n1}+t_{n2}$	$t_{n2}+t_{n3}$

Dispatching

It is concerned with starting the processes. It gives necessary authority to start a particular work, which has already being planned under Routing and scheduling. For starting the work, essential orders and instructions are given. Therefore, the complete definition of dispatching →

"Released of order and instructions for the starting of production for any item in accordance with the route sheet and scheduled chart."

Function of Dispatching

1. After dispatching is done, required materials are moved from stores to m/c(s) and from operation to operation.
2. Authorizes to take work in hand as per schedule.
3. To distribute m/c loading and schedule charts route sheets and other necessary instructions and forms.

4. To issue inspection orders, clearly stating the type of inspection required at various stages.
5. To order too section for issuing proper tools, jigs, fixtures and other essential articles.

Forms used in Dispatching

Following are some of the more common forms used in dispatching.

(a) Work orders: while starting the production, work orders are issued to departments to commence the desired lot of product.
(b) Time cards: Each operator is supplied with this card in which he mentions the time taken by each operation and other necessary information's. there are helpful for wage payment.
(c) Inspection Tickets: These tickets are sent to the inspection department which shows the quality of work required and stages at which inspection is to be carried out.
(d) Move Tickets: These tickets are used for authorizing over the movement of material from store to shop and from operation to operation.
(e) Tool & Equipment Tickets: It authorizes the tool department that new tools, gauges, jigs, fixtures and other required equipment may be issued to shop.

CHAPTER – 2: INVENTORY CONTROL

© 1. Introduction

Inventory is defined as the list of movable goods which are necessary to manufacture a product and to maintain the equipments and machinery in good working order/condition.

Classification

Broadly Classified into

- Direct inventory
- Indirect inventory

i. **Direct inventory**

It plays direct role in the manufacture of product such as:

- Raw materials
- Inprocess inventories (= work in progress)
- Purchased parts (purchasing of some components instead of manf. in the plant)
- Inished goods.

ii. **Indirect inventory**

it helps the raw materials to get converted into finished part. such as:

- Tools
- Supplies
  - miscellaneous consumable – brooms, cotton, wool, jute, etc.
  - welding electrode, solders etc.
  - abrasive mat – emery cloth, sand paper etc.
  - brushes, maps, etc.
  - oil greases etc.
  - general office supplies – candles, sealing wax etc.
  - printed forms such as – envelope, letter heads, quotation forms etc.

Inventory control

Inventory control means – making the desired items of required quality and quantity available to various departments/section as & when they need.

(c) Relevant costs

The relevant costs for how much & when decisions of normal inventory keeping one:

1. **Cost of capital**
   Since inventory is equivalent to locked-up working capital the cost of capital is an important relevant cost. this is the opportunity cost of investing in inventory.
2. **Space cost**
   Inventory keeping needs space and therefore, how much and when question of inventory keeping are related to space requirements. this cost may be the rent paid for the space.
3. **Materials handling cost**
   The material need to be moved within the warehose and the factory and the cost associated with the internal movement of materials (or inventory) is called materials handling cost.
4. **Obsolescence, spoilage or Deterioration cost**
   If the inventory is procured in a large quantity, there is always a risk of the item becoming absolute due to a change in product design or the item getting spoiled because of natural ageing process. Such cost has a relation to basic question of how much and when?
5. **Insurance costs**
   There is always a risk of fire or theft of materials. a firm might have taken insurance against such mishaps and the insurance premium paid are the relevant cost.
6. **Cost of general administration**
   Inventory keeping will involve the use of various staffs. with large inventories, the cost of general administration might go up.
7. **Inventory procurement cost**

Cost associated with the procurement activities such as tendering, evaluation of bids, ordering, follow-up the purchase order, receipt and inspection of materials etc. is called inventory procurement cost.

(c) Basic EOQ model

EOQ = Economic Order Quantity.

EOQ represent the size of the order (or lot size) such that the sum of carrying cost (due to holding the inventory) and ordering cost is minimum. it is shown by point A of figure 2.1.

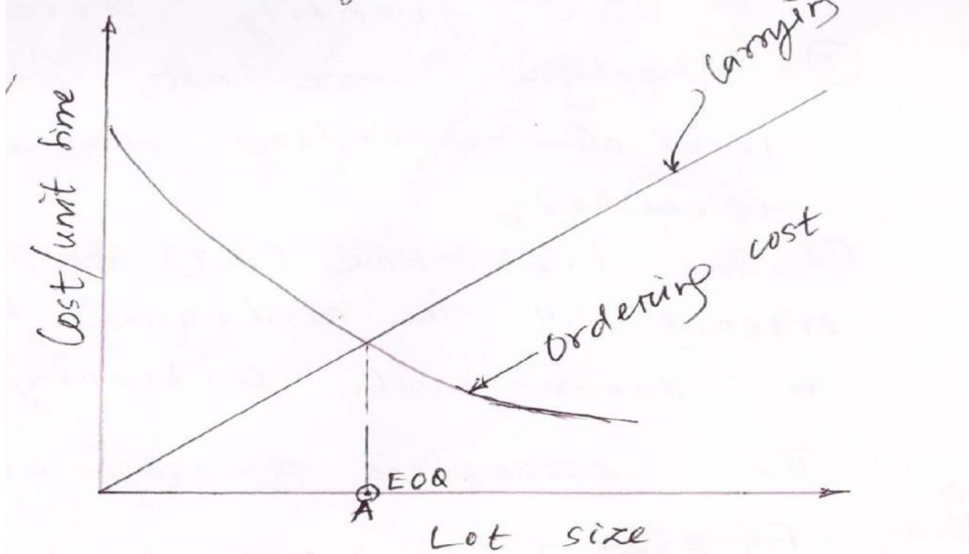


Fig 2.1: Representation of EOQ.

As mentioned earlier, the two most important decisions related to inventory control are:

- When to place an order? &
- How much to order?

In 1913, F.W. Harris developed a rule for determining optimum number of units of an item to purchase based on some fundamental assumptions. This model is called Basic Economic Order Quantity model. it has broad applicability.

Assumptions

The following assumptions are considered for the sake of simplicity of model.

1) Demand (D) is assumed to be uniform.
2) The purchase price per unit (P) is independent of quantity ordered.
3) The ordering cost per order (Co) is fixed irrespective of size f lot.
4) The carrying cost/holding cost (Cc) is proportional to the quantity stored.
5) Shortage are not permitted i.e., as soon as the level of inventory reaches zero, the inventory is replenished.
6) The lead time (LT) for deliveries (i.e. the time of ordering till the material is delivered) is constant and is known with certainty.

The assumptions 5 and 6 are shown graphically in fig 2.2.

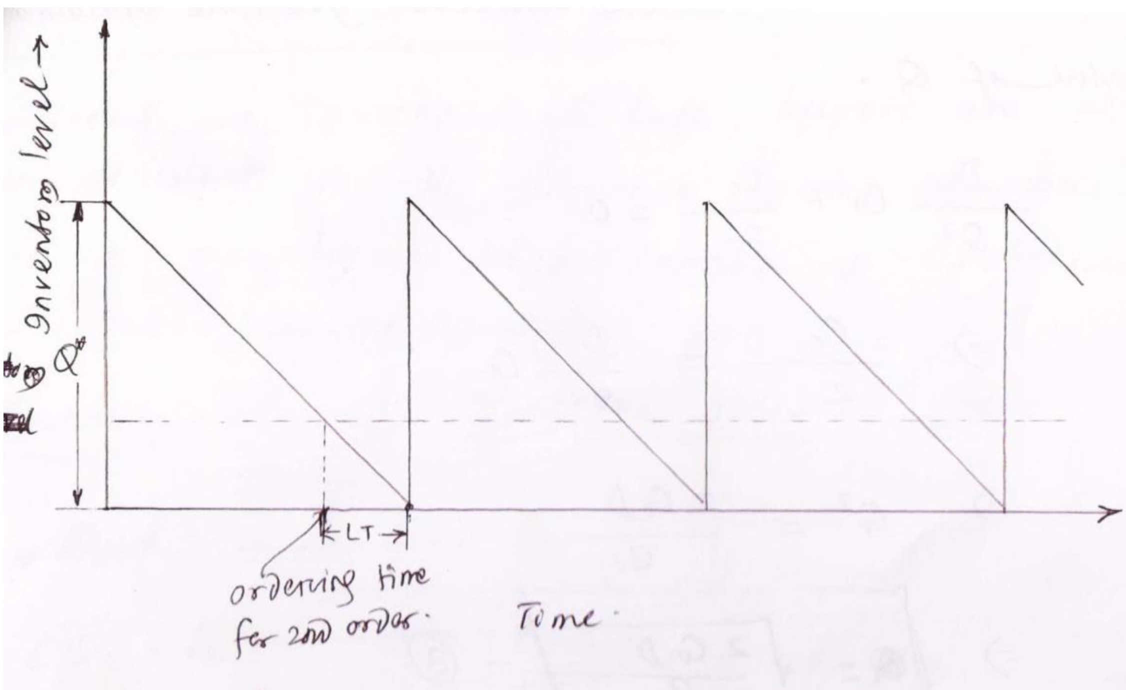


Fig 2.2: Purchase model without stock.

Let Q = order size

Therefore, the number orders/year = $\frac{D}{Q}$------------(1)

Average inventory level = $\frac{Q}{2}$

Ordering cost per year = $\frac{D}{Q} \times Co$ ------------ (2)

Carrying cost per year = $\frac{Q}{2} \times Cc$ ------------- (3)

Purchase cost/year = $D \times P$ --------------- (4)

Now, the total inventory cost per year = $TC = \frac{D}{Q} \times Co + \frac{Q}{2} . \ Cc + D \times P$ ---- (5)

Differentiating Eq (5) w.r.t. Q it becomes:

$\frac{d(TC)}{dQ} = \frac{-D}{Q^2} Co + \frac{Cc}{2}$ ----------- (6)

The 2nd derivative = $\frac{+2D}{Q^3} Co$ --- (7)

Since the 2nd derivative is +ve, we can equate the value of first derivative to zero to get the optimum value of Q.

i.e., $\frac{-D}{Q^2} Co + \frac{Co}{2} = 0$

:- $\frac{Cc}{2} = \frac{D}{Q^2} Co$

:- $Q^2 = \frac{2CoD}{Cc}$

:- $Q = \sqrt{\frac{2CoD}{Cc}}$ ------ (8)

So, optimum $Q = EOQ = \sqrt{\frac{2CoD}{Cc}}$

Ex: ABC company estimates that it will sell 12000 units of its product for the forthcoming year. the ordering cost is Rs 100/- per order and the carrying cost per year is 20% of the purchase price per unit. The purchase price per unit is Rs 50/-.

Find

i. Economic Order Quantity
ii. No. of orders/year
iii. Time between successive order.

Solution:

Given D = 12000 units/yr, Co = Rs 100/year

Cc = Rs 50×0.2 = Rs 10/- per unit/year

Therefore (i) $EOQ = \sqrt{\frac{2CoD}{Cc}} = \sqrt{\frac{2\times100\times12000}{10}} = 490$ units approx.

No. of orders/year = $\frac{D}{Q*} = \frac{12000}{490} = 24.49$

Time between successive order = $\frac{Q*}{D} = \frac{490}{12000} = 0.04$ year $= 0.48$ month

© **Models with Quantity Discount**

When items are purchased in bulk, buyers are usually given discount in the purchase price of goods. this discount may be a step function of purchase quantity as stated in the Following.

Quantity	Purchase price
$0 \leq Q_1 < b_1 \rightarrow$	P_1
$b_1 \leq Q_2 < b_2 \rightarrow$	P_2
:	:
:	:
$b_{n-1} \leq Q_n \rightarrow$	P_n

The procedure to compute the optimal order size for this situation is given in the following steps.

Step- 1

Find EOQ for nth (last) price break

$$Q^*_n = \sqrt{\frac{2CoD}{iPn}}$$

Where i = fraction of purchase price for inventory carrying.

If it is greater than or equal so b_{n-1}, then the optimal order size Q = Q_n; otherwise go to step-2.

Step- 2

Find EOQ for (n-1)th price break

$$Q^*_{n-1} = \sqrt{\frac{2\ \ CoD}{i\ Pn{-}1}}$$

If it is greater than or equal to b_{n-2}, then compute the following and select the least cost purchase quantity as optimal order size; otherwise go to step-3

i) TC (Q_{n-1})
ii) TC (b_{n-1})

Step-3

Find EOQ for (n-2)th price break

$$Q^*_{n-2} = \sqrt{\frac{2\ \ CoD}{I\ \ Pn{-}2}}$$

If it d greater than or equal to bn-3, then compute the following and select the least cost purchase quantity; otherwise go to step-4.

i) Total cost, TC (Q^*_{n-2})
ii) Total cost, TC (b_{n-2})
iii) Total cost, TC (b_{n-1})

Step-4

Continue in this manner until $Q^*_{n-k} \geq b_{n-k-I}$. Then compare total cost $\pi(Q^*_{n-k})$, $\pi(b_{n-k})$, $\pi(b_{n-k+1})$………….. $\pi(b_{n-1})$ corresponding to purchase quantities

Q^*_{n-k}, b_{n-k}, b_{n-k+1}, ……….b_{n-1} respectively. Finally select the purchase quantity w.r.t. minimum total cost.

Ex: Annual demand for an item is 4800 units. ordering cost is Rs 500/- per order. inventory carrying cost is 24% of the purchase price per unit. the price break are given below.

Quantity		Price
$0 < Q_1 < 1200(b_1)$	→	10
$1200 \le Q_2 < 2000(b_2)$	→	9
$2000 \le Q_3$ (b_3)	→	8

(a) Find optimal order size.

Solution (a) D = 4800, Co = 500, I = 0.24

Step- 1 P_3 = Rs 8/- $Q_3 = \sqrt{\frac{2\ CoD}{i\ P3}} = \sqrt{\frac{2\times500\times4800}{0.24\times8}} = 1581$

Since Q3 < b2, i.e., 2000, go to step-2

Step- 2 P_2 = Rs 9/- $Q_2 = \sqrt{\frac{2\ CoD}{i\ P2}} = \sqrt{\frac{2\times500\times4800}{0.24\times9}} = 1491$

Since $Q_2 > b_1$ i.e., 1200, find the following costs & select the order size based on least cost.

$$TC(Q_2) = 9 \times 4800 + 500 \times \frac{4800}{1491} + \frac{0.24\times9\times1491}{2} = \text{Rs } 46420 \text{ (approx)}$$

$$TC\ (b_2) = 8 \times 4800 + 500 \times \frac{4800}{2000} + \frac{0.24\times8\times2000}{2} = \text{Rs } 41{,}520 \text{ (approx)}$$

The least cost is Rs 41,520, Hence optimal order size is 2000.

©Economic Batch Quantity

(a) Without shortage.

It is a manufacturing model

(b) With shortage.

(a) Manufacturing model without shortage

if a company manufacture its component which is required for its main product, then the corresponding model of inventory is called ⍰ manufacturing model. This model will be without/with shortage. The rate of consumption of item is uniform throughout the year. The cost of production per unit is same irrespective of production 10+ size. Let,

r = annual demand of an item

k = production rate of item (No. of units produced per year)

Co = cost per set-up.

Cc = carrying cost per unit per period.

P = cost of production per unit

EOQ = Economic Batch Quantity

The variation of inventory with time without shortage is shown below.

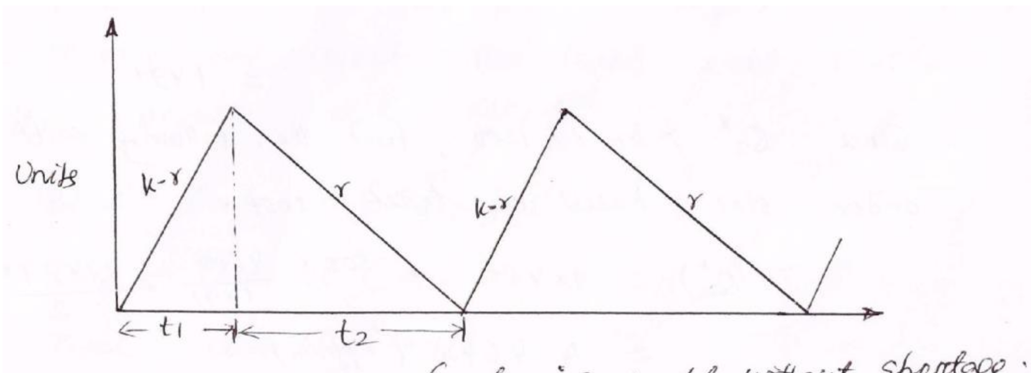


Fig 2-3: Manufacturing model without shortage.

During the period t_1, the item is produced at the rate of k units per period and simultaneously it is consumed at the rate of r units per period. So during this period, the inventory is built at the rate of (k-r) units per period.

During the period t2, the production of items is discontinued but the consumption of item is continued. Hence the inventory is decreased at the of r units per period during this period.

The various formulae for this situation

 Economic Batch Quantity (EBQ) = $\sqrt{\frac{2\ Cor}{Cc\ (1-r/k)}}$

$t1^* = Q^*/K$

$t2^* = \frac{Q*[\,1-r/k]}{r}$

Cycle time = t1* + t2* (Refer Operation Research Book by Kanti Swarup for detail)

Ex: If a product is to be manufactured within the company, the details are as follows:

r = 24000 units/year

k = 48000munits/year

Co = Rs 200/- per set-up

Cc = Rs 20/- per unit/year

Find the EBQ & Cycle time

Solution:

$$EBQ = \sqrt{\frac{2\ Co.\ r}{Cc\ \ (1-r/k)}} = \sqrt{\frac{2\times200\times24000}{20(\,1-24000/48000)}} = 980\ approx.$$

$$t^*_1 = \frac{Q*}{k} = \frac{980}{48000} = 0.\ 02yr\ = 0.\ 24\ \ mokt\,h$$

$$t^*_2 = \frac{Q*}{r}(1 - \frac{r}{k}) = \frac{980}{24000}(1 - \frac{24000}{48000}) = 0.\ 02yr\ = 0.\ 24\ \ mokt\,h$$

total cycle time = $t^*_1 + t^*_2$ = 0.24+0.24 = 0.48 month

(b) Manufacturing model with shortage

In this model, the items are produced and consumed simultaneously for a portion of cycle time. The rate of consumption of items is uniform through out the year. The cost of production per unit is the same irrespective of production lot size. In this model, stock out/shortage is permitted. It is assumed that the stock out units will be satisfied from the units which will be produced at a later date with a penalty (like rate reduction). This is called back ordering. The operation of this model is shown in fig. 2.4. .

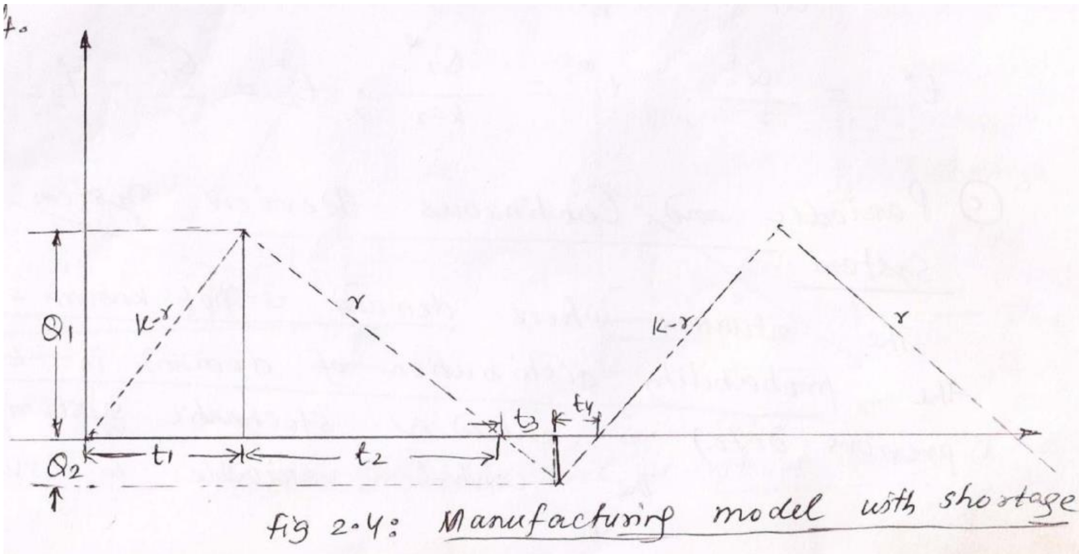


fig 2.4: Manufacturing model with shortage

The variables which are used in this model are given below.

r = annual demand of an item

k = production rate of the item

Co = cost/setup

Cc = carrying cost/unit/period

Cs = shortage cost/unit/period.

P = cost of production per unit

In the above model

Q = Economic batch quantity

Q1 = Maximum inventory

Q2 = Maximum stock out

By applying mathematics

$$Q^* = EBQ = \sqrt{\frac{2\,Co}{Cc}\,\frac{kr}{(k-r)}\,\frac{(Cc+Cs)}{Cs}} \text{ ----------(1)}$$

$$Q^*1 = \sqrt{\frac{2\,Co\,r}{Cc}\frac{(k-r)}{k} \times \frac{Cs}{Cc+Cs}} \text{ --------------(2)}$$

$$Q^*2 = \sqrt{\frac{2\ \ Co\ \ Cc}{Cs(Cc+Cs)} \times \frac{r(k-r)}{k}}$$

$$\text{Also } Q^*1 = \left(\frac{k-r}{k}.\ Q\right) - Q^*_2$$

$$t^* = \frac{Q^*}{r};\ t^*_1 = \frac{Q^*_1}{k-r};\ t^*_2 = \frac{Q^*_1}{r};\ t^*_3 = \frac{Q^*_2}{r};\ t^*_4 = \frac{Q^*_2}{(k-r)}$$

(c) Periodic and Continuous Review system for stochastic system (= probabilities)

The situation where demand is not known exactly but the probability distribution of demand is known (from previous data) is called a stochastic system/problem.

The control variable in such case is assumed to be either

- The scheduling period. or
- The order level. or
- Both.

The optimum order level will thus be derived by minimizing the total expected cost rather than the actual cost involved.

Stochastic problem with uniform demand

The following assumptions are made for the simplicity of model.

1) Demand is uniform over a period (let r unit/period)
2) Reorder time is fixed and known
3) Production of commodities is instaneous, and
4) Lead time is negligible.

Let (i) the holding cost/carrying cost per unit item = Cc

(ii) the shortage cost/item/time = Cs

(iii) the inventory level at any time t = Q

The problem is to determine the optimum order level Q (without shortage) at the beginning of each period, where Q ≥ r or Q < r (with shortage). In both these cases the inventory system is shown in fig. 2.5 below.

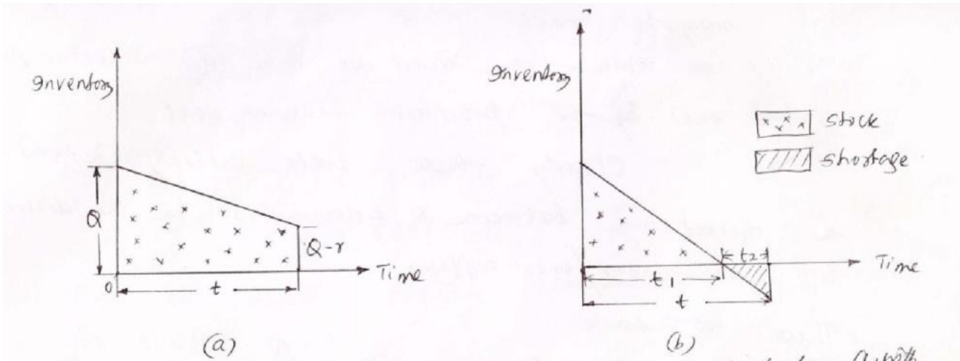


Fig 2.5 : Inventory level (a) - without shortage (b) with Shortage.

The units that build up an inventory may consist of either discrete (= periodic) /continuous system.

(A) Periodic system (or Discrete unit)

Let the demand for r unit be estimated at a discontinuous rate with probability P(r); r = 1,2,3, ...n. That is we may expect demand for 1 unit with probability, p(1); 2 units with probability p(2); and so on. Since all the possibilities are taken care of, we must have

$$\sum_{r=1}^{\infty} P(r) = 1akd \quad P(r) \geq 0$$

Penalty costs are associated with producing Q which is less than the amount actually demanded (i.e. Q < r). It is denoted by the shortage cost (Cs). This may be made up of either

i. Loss of good will &
ii. Contract penalty for failure to deliver.

Similarly we assume that the penalty costs are associated with producing Q, which is lying surplus even after meeting the demand (i.e. Q $\geq r$). We denote this cost by Cc, as unit cost of oversupplying. This may be made up of either

i) Loss, when extra items are to be sold at lesser price. &
ii) Held by the producer incurring cost.

Clearly these costs entirely depend upon the discrepancy between Q & demand (r). And then discrepancy is an under/oversupply.

Expected size of over supply = $\sum_{r=1}^{Q} (Q - r) P(r)$ ———(1)

Expected cost of over production = $Cc \sum_{r=1}^{Q} P(r)$---------(2)

Expected size of undersupply = $\sum_{r=Q+1}^{\infty} (r - Q).\ P(r$-)---(3)

Expected cost of undersupply = Cs $\sum_{r=Q+1}^{\infty} (r - Q) P(r)$---(4)

Thus the total expected cost

TEC(Q) = Cc $\sum_{r=1}^{Q} (Q - r).\ P(r) + Cs \sum_{r=Q+1}^{\infty} (r - Q).\ P(r$-)---(5)

The problem now is to find Q so as to minimize TEC(Q).

Let on amount Q+1 instead of Q be produced. Then the total expected cost equation →

$TEC(Q) = Cc \sum_{r=1}^{Q+1} (Q + 1 - r).\ P(r) + Cs \sum_{r=Q+2}^{\infty} (r - Q - 1) P(r)$-----(6)

On simplification (referring O.R. by Kanti Swarup)

TEC(Q+1) = TEC(Q)+(Cc+Cs) $\sum_{r=1}^{Q} P(r) - Cs$

& TEC(Q+1) = TEC(Q)+(Cc+Cs) P(r ≤ Q)-Cs -----------(7)

Similarly, when an amount Q-1, instead of Q is produced,

TEC(Q-1) = TEC(Q)-(Cc+Cs) P(r ≤ Q-1)+Cs ---------------(8)

Suppose that we find Q* having the property that

(i) TEC(Q*) < TEC(Q*+1) &
(ii) TEC Q* < TEC (Q*-1),

Then Q* would clearly represent a local minimum for TEC(Q)

Let us define ΔTEC(Q) = TEC(Q+1) – TEC(Q) as the difference between the total expected cost for Q and for the next higher value (Q+1). Thus from Eq(7) & (8), we have

Δ[TEC(Q)] = (Cc+Cs)P(r ≤ Q) - Cs

And Δ[TEC(Q-1)] = (Cc+Cs) P(r ≤ Q-1) – Cs

Therefore, if Q* be the local minima for TEC(Q), then

(i) TEC(Q*) < TEC(Q*+1) :- Δ[TEC(Q*)] > 0

:- (Cc+Cs) P(r ≤ Q) – Cs > 0

:- P (r ≤ Q) > $\frac{Cs}{Cc+Cs}$ ---------------------(9)

And (ii) TEC(Q*) < TEC(Q*-1) :- Δ[TEC(Q*-1)] <0

:- (Cc+Cs) P(r ≤ Q-1)-Cs < 0

:- P(r ≤ Q-1) < $\frac{Cs}{Cc+Cs}$------------- (10)

From Eq (9) & (10)

$$P(r \leq Q-1) < \frac{Cs}{Cc+Cs} < P(r \leq Q)$$

Hence if the oversupply cost Cc and the shortage cost Cs are known, the optimal quantity Q* is determined when the value of cumulative probability distribution P (r) just exceeds the ratio $\frac{Cs}{Cs+Cs}$. That is Q* is determined by comparing a cost ratio with probability figure.

(B) <u>continuous Review System</u>

When certain demand estimated as a continuous random variable, the cost equation of the inventory involves integrals instead of summation sign. The discrete point probabilities p(r) are replaced by probability differential f(r) for small interval, say $r \pm \frac{dr}{2}$ of continuous demand variable. In this case $\int_1^\infty f(r)dr = 1$ and f(r) ≥ 0. Proceeding exactly in the manner as (A), Let

Q = quantity produced

Cc = penalty cost per unit cost of oversupply (Q ≥ r), and

Cs = penalty cost per unit cost of under supply (Q < r)

The expected sizes of over and under supply are:

$\int_{r=1}^{Q}(Q-r)F(r)dr \quad akd\int_{r=Q}^{\infty}(r-Q)f(r)dr$ respectively

The total expected cost (TEC) associated with producing an amount Q when facing a demand known only as a continuous random variable is given by:

$$TEC(Q) = C_c\int_{r=1}^{Q}(Q-r)F(r)dr + Cs\int_{Q}^{\infty}(r-Q)f(r)dr \text{------}(11)$$

We now determine optimum value Q* so as to minimize TEC (Q)

$$\frac{\partial TEC(Q)}{\partial Q} = \frac{\partial}{\partial Q}\{Cc\int_{1}^{Q}(Q-r)f(r)dr + Cs\int_{Q}^{\infty}(r-Q)f(r)dr\}$$

$$= Cc[\int_{1}^{Q}f(r)dr + (Q-Q).1-(Q-1)f(1).0] + Cs[-\int_{Q}^{\infty}(f(r)dr + (r-Q)f(r)\frac{dr}{dQ}/-(Q-Q)f(Q).1]$$

$$= Cc\int_{1}^{Q}f(r)dr - Cs\int_{Q}^{\infty}f(r)dr$$

$$= Cc\int_{1}^{Q}f(r)dr - Cs[\int_{1}^{\infty}f(r)dr - \int_{1}^{Q}f(r)dr]$$

$$= (Cc+Cs)\int_{1}^{Q}f(r)dr - Cs \qquad \text{since } \int_{1}^{\infty}f(r)dr = 1$$

$$\rightarrow \frac{6TEC(Q)}{6Q} = 0 = (Cc+Cs)\int_{1}^{Q}f(r)dr = Cs = \int_{1}^{Q}f(r)dr = \frac{Cs}{Cc+Cs}$$

$$\& P(r \leq Q) = \frac{Cs}{Cc+Cs} \text{------------------}(12)$$

Moreover,

$$\frac{6^2TEC(Q)}{6Q^2} = (Cc+Cs)f(Q) > 0$$

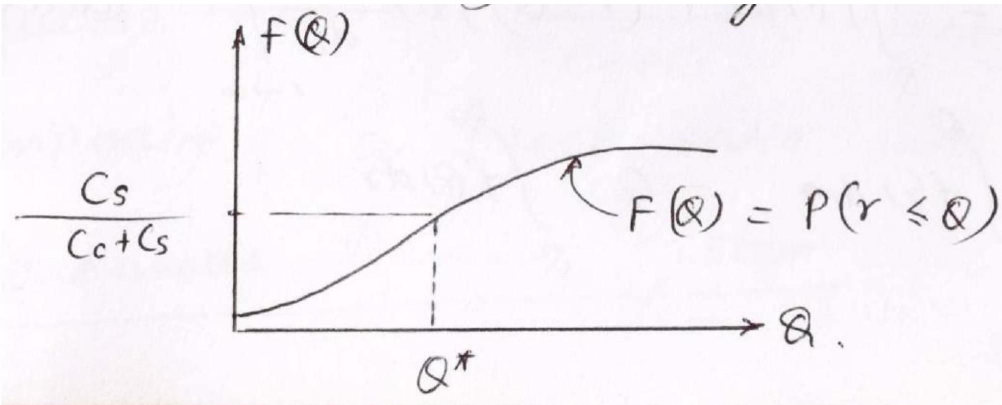
Thus Q as determined by Eq (12) is an optimal value so as to minimize TEC(Q).

$$\text{Hence } P(r \leq Q) = F(Q^*) = \frac{Cs}{Cc+Cs} \text{----------}(13)$$

Where $F(Q) = \int_{1}^{Q}f(r)dr$

This indicates that "the best quantity to be produced is that value of Q for which the value of cumulative probability distribution of r is equal to $\frac{Cs}{Cc+Cs}$".

The optimum value of Q for continuous demand variable may be illustrated graphically as shown below.



Ex: A newspaper boy buys paper for Rs 1.40 and sells them for Rs 2.45. He can't return unsold newspaper. Daily demand has the following distributions.

Number of customers	25	26	27	28	29	30	31	32	33	34	35	36
Probability	0.03	0.05	0.05	0.10	0.15	0.15	0.12	0.10	0.10	0.07	0.06	0.02

If each days demand is independent of the previous days, how many papers he should order each days?

Solution:

Given Cc = Rs1.40, Cs = Rs2.45 – 1.40 = Rs1.05

The point probabilities are:

r	25	26	27	28	29	30	31	32	33	34	35	36
P(r)	0.03	0.05	0.05	0.10	0.15	0.15	0.12	0.10	0.10	0.07	0.06	0.02
∑ P(r)	0.03	0.08	0.13	0.23	0.38	0.53	0.65	0.75	0.85	0.92	0.98	1.0

Now $\frac{Cs}{Cc+Cs} = \frac{1.05}{1.40+1.05} = 0.4285$

Now 0.38 < 0.4285 < 0.53 ; so the number of newspaper ordered = 30.0

(C) Safety stock, Recorder point and Order Quantity Calculation

The safety stock may be defined as minimum aditional inventory to serve as a safety margin (or cushion) to meet unanticipated increase in usage resulting from various uncontrollable factors like

i) An unusual high demand
ii) Late receipt of incoming inventory

The reorder level (ROL) = DLT+ safety stock (SS)

Where DLT = Demand during lead time

= Demand rate × Lead time period (from geometry)

$= \frac{Demand}{day} \times LT$ in days.

Safety Stock (SS) = kσ where k = standard normal statistic value for a given service level & σ = standard deviation.

Ex: A firm has a demand distribution during a constant lead time with a standard deviation of 250 units. The firm wants to provide 98% service

a) How much safety stock should be carried.
b) If the demand during lead time averages 1200 units, what is the appropriate recorder level. (ROL)?

Corresponding to 98% service level, K valve from normal distribution table = 2.05.

Solution:

a) Safety stock (SS) = kσ = 2.05 × 250 = 512 units
b) ROL = D_{LT} + SS = 1200+512 = 1712 units

(C) ABC Analysis

ABC means → Always Better Control

ABC analysis divides inventories into three groupings in terms of percentage of number of items and percentage of total value. in ABC analysis important items (high usage valued items) are grouped in C and the remaining middle level items are considered 'B' items.

The inventory control is exercised on the principle of "management by exception" i.e., rigorous controls are exercised on A items and routine loose controls for C items and moderate control in 'B' items. The items classified by virtue of their uses as:

Category	% of items (approx)	% value (approx)
A – High value items	10	70
B – Medium value items	20	20
C – Low value items	70	10

Control policies for A items

i) 'A' items are high valued items hence should be ordered in small quantities in order to reduce capital blockage.

ii) The future requirement must be planned in advanced so that required quantities arrive a little before they are required for consumptions.

iii) Purchase and stock control of A items should be taken care by top executives in purchasing department.

iv) Maximum effort should be made to expedite the delivery.

v) The safety stock should be as less as possible (15 days or less).

vi) 'A' items are subjected to tight control w.r.t.
- Issue
- Balance
- Storing method

vii) Ordering quantities, reorder point and maximum stock level should be revised more frequently.

Control policies for 'C' items

i) The policies for 'B' items are in general between A & C.

ii) Order for these items must be placed less frequently.

iii) Safety stock should be medium (3 months or less).

iv) 'B' items are subjected to moderate control.

Control policies for 'C' items

i) 'C' items are low valued items.
ii) Safety stock should be liberal (3 months or more).
iii) Annual or 6 monthly order should be placed to reduce paper work & ordering cost and to get the advantage of discount.
iv) In case of these items only routine check is required.

Steps for ABC Analysis

1. Calculate the annual usage in units for each items.
2. Calculate the annual usage of each item in terms of rupees.
3. Rank the items from highest annual usage in rupees to lowest annual usage in rupees.
4. Compute total rupees.
5. Find the % of high, medium and low valued items in terms of total value of items.

The following example will give a clear and wide information about ABC analysis. Prepare ABC analysis on the following sample of items in an inventory.

Item	Annual usage unit	Unit cost (Rs)	Annual usage (Rs)	Ranking
a	30,000	0.01	300	6
b	2800	1.5	4200	1
c	300	0.10	30	9
d	1100	0.5	550	4
e	400	0.05	20	10
f	2200	1.0	2200	2
g	1500	0.05	75	8
h	8000	0.05	400	5
i	3000	0.30	900	3
j	800	0.10	80	7

Table showing ABC Analysis (ABC Ranking)

Item	Annual usage (Rs)	Cumulative amount	Cumulative %	Ranking	Annual usage units	Cumulative %
b	4200	4200	47.97	A	2800	5.88

f	2200	6400	73.10	A	2200	9.98
i	900	7300	83.38	B	3000	15.97
d	550	7850	89.66	B	1100	18.16
h	400	8250	94.23	B	8000	34.13
a	300	8550	97.66	C	30,000	94.01
j	80	8630	98.57	C	800	95.68
g	75	8705	99.43	C	1500	98.60
c	30	8735	99.77	C	300	99.20
e	20	8755	100.0	C	400	100

Accordingly a graph can be plotted.

Benefits of ABC Analysis (By a suitable example)

A company that has not made ABC analysis of its inventory makes 4 orders/year in respect of each item to get 3 months supply of every item. Taking a sample of 3 items, with different levels of annual consumptions, their average inventory (which is one half of order quantity) is worked out in the following table.

Item	Annual consumption	No. of orders	Average working inventory
A	40,000	4	$\frac{10,000}{2} = 5000$
B	4000	4	$\frac{1000}{2} = 500$
C	400	4	$\frac{100}{2} = 50$
Total		**12**	**5550**

But keeping the same no. of orders/year (i.e. 12), inventory can be reduced by 39% by segregating them according to their usage value (ABC analysis) as illustrated in the following table.

Item	Annual consumption	No. of orders	Average working inventory
A	40,000	8	$\frac{5000}{2} = 2500$
B	4000	3	$\frac{1333}{2} = 667$
C	400	1	$\frac{400}{2} = 200$
Total		**12**	**3367**

Thus the investment on inventory is reduced.

Applications of ABC analysis

ABC analysis can be effectively used in materials management. Such as

- Controlling raw materials components.
- Controlling work in progress inventories.

Limitations of ABC analysis

1. ABC analysis does not consider all relevant problems of inventory control such as a firm handling adequately low valued 'C' items.
2. ABC analysis is not periodically revised for which 'C' items like diesel oil in a firm will become most high valued items during power crisis.
3. The importance of an item is computed based on its consumption value and not its criticality.

MODULE – II CHAPTER- 3: PROJECT MANAGEMENT

1. What is a project?
    a. Project is an interrelated set of activities that has a definite starting and ending point resulting in to an unique product. few examples of project are-
        - Constructing a bridge, dam, highway or building.
        - Producing an aeroplane, missile or rocket.
        - Introducing a new product.
        - Installation of a large computer system.
        - Construction of a ship.
        - Maintenance of major equipment/plant.
        - Commissioning of a power plant.
        - Conducting national election.
2. Basic steps in project management

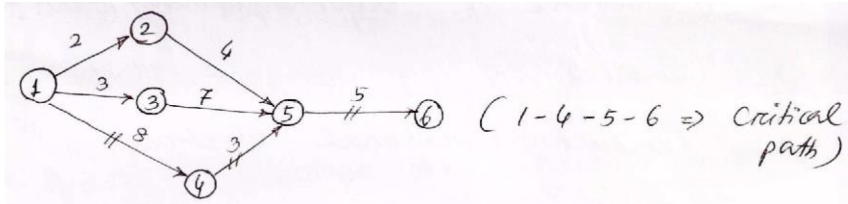
    Managing a project (regardless of its size and complexity) requires identifying every activity to be undertaken and planning- when each activity must begin and end in order to complete the overall project in time. typically all projects involve the following steps:
    - Description of the project.
    - Development of network diagram.
    - Insertion of time of starting/ending of each activity.
    - Analysis of the network diagram.
    - Development of the project plan
    - Excetion of the project.
    - Periodically assessment of the progress of project.
3. Terminologies used in network diagram

(i) **Activity:** An activity means work/job. it is a time consuming process. it is represented by an arrow (→) in the network diagram. as shown below.

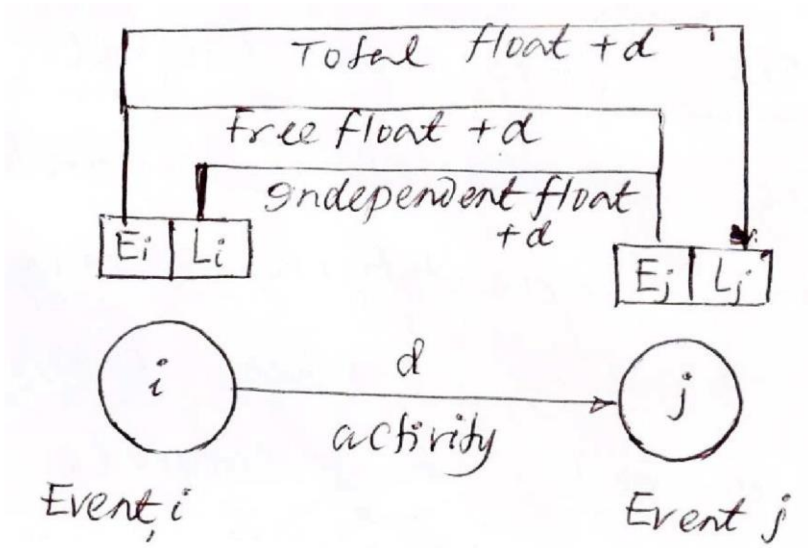
Tail → head

(ii) **Event :** An event is a specific instant of time marks the "start" and "end" of an activity.

(iii) **Critical path:** It is the sequence of activities which decides the total project duration. Ex.



(iv) **Duration (d)** : Duration is the estimated or actual time required to complete a task or an activity.



(v) **Total project time:** time to complete the project. In other words, it is the duration of critical path.

(vi) **Earliest start time (Ei)**: It is the earliest possible time at which an activity can start. it is calculated by moving from 1st to last event in the network diagram.

(vii) **Latest start time (Li)** : It is the latest possible time by which an activity can start.

(viii) **Earliest finish time (Ej)** : It is the earliest possible time at which an activity can finished/end.

(ix) **Latest finish time (Li)** : It is the last event time of the head event. It is calculated by moving backward in the network diagram.

(x) **Float/slack** : Slack is with reference to an event. Float is with reference to an activity.

(xi) **Total float** : (Latest finish time- Earliest start time) - Activity duration (Su fig. above)

(xii) **Free float** : (Earliest finish time- Earliest start time) – Activity duration.

(xiii) **Independent float** : (EST of head event – LST of tail event) – Activity duration.

(xiv) **Optimistic time (to)** : Time estimate for fast activity completion.

(xv) **Pessimistic time (tp.)** : Maximum time duration. that an activity can take.

(xvi) **Most likely time (tm)** : best guess of activity completion time.

(xvii) **Expected time (te)** : $\frac{to+4tm+tp}{6}$

(xviii) **Variance of an activity time** : $6_e^2 = (\frac{tp-to}{6})^2$

(xix) **CPM** – Critical path method

(xx) **PERT-** Program evaluation & review technique.

© Project management through PERT/CPM.

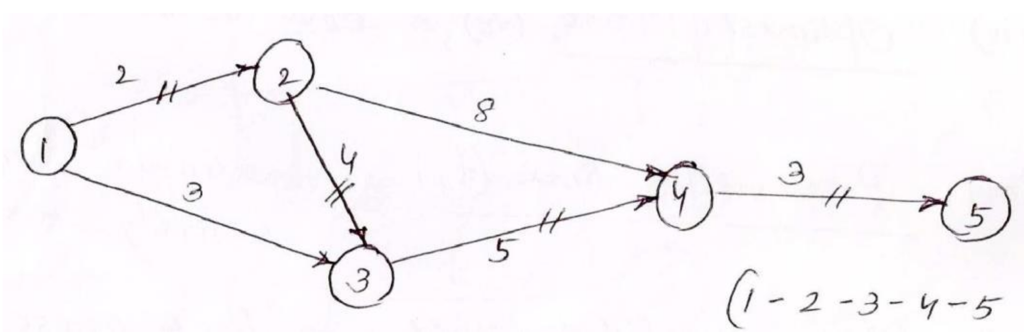
Project scheduling using Gantt chart was done from 1917. Till 1956. Between 1956-58 two new scheduling techniques were developed.

(i) PERT

(ii) CPM

Both are based on the use of a network/graphical model to depict the work tasks being scheduled. The popularility of network based scheduling can be attributed to its many benefits, especially its ease of use. Other benefits include the following.

(1) It provides a visual display of needed task and their temporal ordering which makes it easy to see how the tasks should be sequenced as shown below.



This assist communication and cooperation among task teams because each team can see how its work affect other team.

(2) It provides relatively accurate estimate of the time required to complete the project at the proposed resource level.

(3) It identifies and highlights the tasks that are critical to keep the project on schedule.

(4) It provides a method for evaluating the time-cost trade-offs resulting from reallocating resources among tasks.

(5) It provides a method for monitoring the project throughout its life cycle. as the project progresses, PERT/CPM easily identifies changes in which tasks are critical and how the expected completion date is affected.

(6) It provides a convenient method for incorporating uncertainty regarding task times into the schedule and it helps to evaluate the effect of this uncertainty on project completion time.

Differences bet PERT & CPM

Sl No.	PERT	CPM
1.	PERT is a probabilistic model with uncertainty in activity duration. activity duration is calculated from t_o, t_p & t_m by relation $t_e = \frac{to+4fm+tp}{6}$	1. CPM is a deterministic model with well known activity duration
2.	It is an event oriented approach	2. It is an activity oriented approach
3.	PERT terminology uses word like network diagram event and slack	3. CPM terminology sues word like arrow diagram nodes and floats
4.	The use of dummy activity is required for representing the proper sequencing	4. No dummy activity
5.	PERT basically does not demarcate between critical and noncritical activity	5. CPM maks the critical activity
6.	PERT is applied in projects where resources are always available	6. CPM in applied to projects where minimum overall cost is the prime importance.
7.	PERT is suitable in defence project & R &D, where activity time can't be readily predicted	7. Suitable for plant maintenance, civil construction projects etc. where activity duration is known.

Rules for Network Construction

1. The network should have a unique starting node (tail event) and unique completion node (head event).
2. No activity should be represented by more than one arrow (→) in the network.
3. No two activities should have the same starting node and same ending node.
4. Dummy activity is an imaginary activity indicating precedena relationship only. duration of dummy activity is zero.
5. The length of arrow bear no relationship to activity time.
6. The arrow in a network identifies the logical condition of dependence.
7. The direction of arrow indicates the direction of work flow.
8. All networks are constructed logically or based on the principal of dependency.
9. No event can be reached in a project before the completion of precedence activity.
10. Every activity in the network should be completed to reach the objective.
11. No set of activities should form a circular loop.

- Network construction, CPM network calculation. (with the help of diff. types of problems)

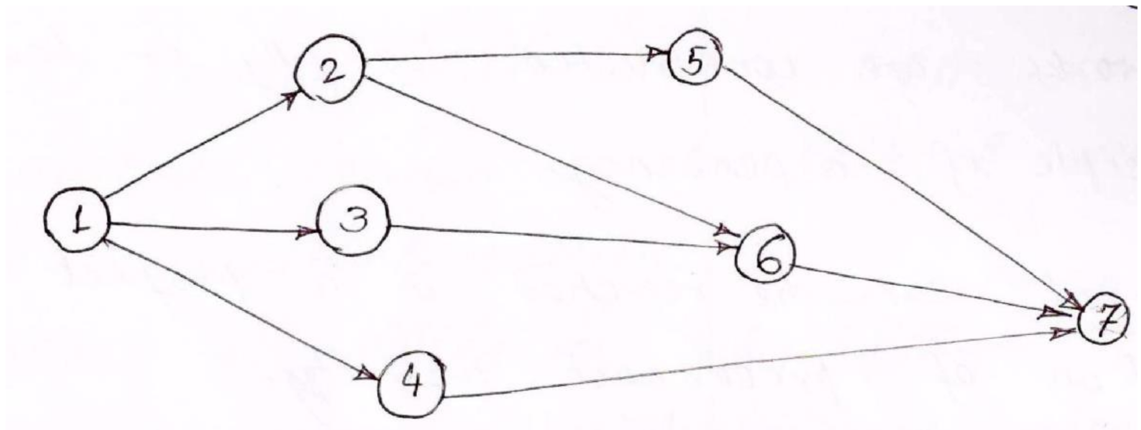
Prob- 1: A project consists of the following activities and time estimates

Activity	Least time/optimistic time (to), days	Greatest time/Pessimistic time (tp), days	Most likely time (tm), days
1-2	3	15	6
1-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

Construct the network. determine the expected task time. show the critical path. what is the project duration?

Solutions

The network diagram is presented in the following

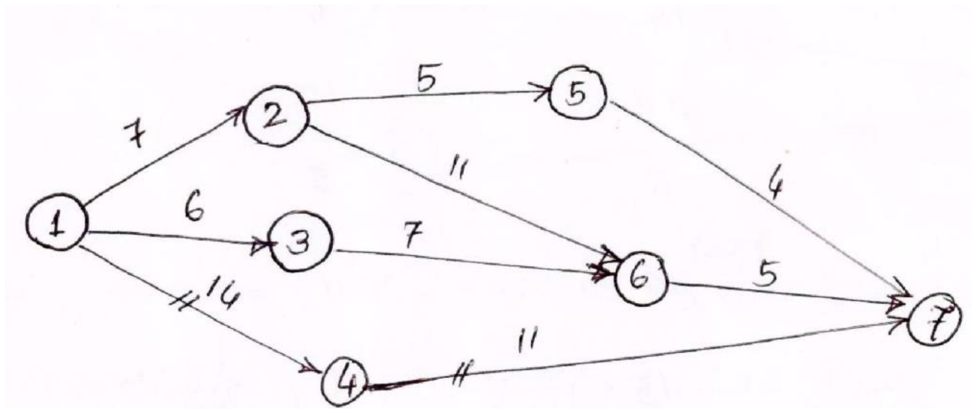


The formula for expected task time (t_e) = $\frac{t_0+t_p+4t_m}{6}$

Accordingly the expected task times for different activities are as follows:

Activity		(t_e) values, days
1-2	→	7
1-3	→	6
1-4	→	14
2-5	→	5
2-6	→	11
3-6	→	7
4-7	→	11
5-7	→	4
6-7	→	5

The expected task times are shown on the network diagram to determined the critical path & project duration.



From this network diagram 1-4-7 represents the critical path and the duration for project completion = total time along the critical path = 14+11 = 25 days

Problem-2

A project consists of the following activities and time estimates.

Activity	Optimistic time (t_0), day	Pessimistic time (t_p), day	Most likely time (t_m), day
1-2	3	15	6
1-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

a) What is the project duration?
b) What is the probability that the project will be completed in 27 days?

Solution:

In order to find out the project duration, the expected task time for each activity is to be determined using the formula → $t_e = \frac{t0+tp+4\ tm}{6}$

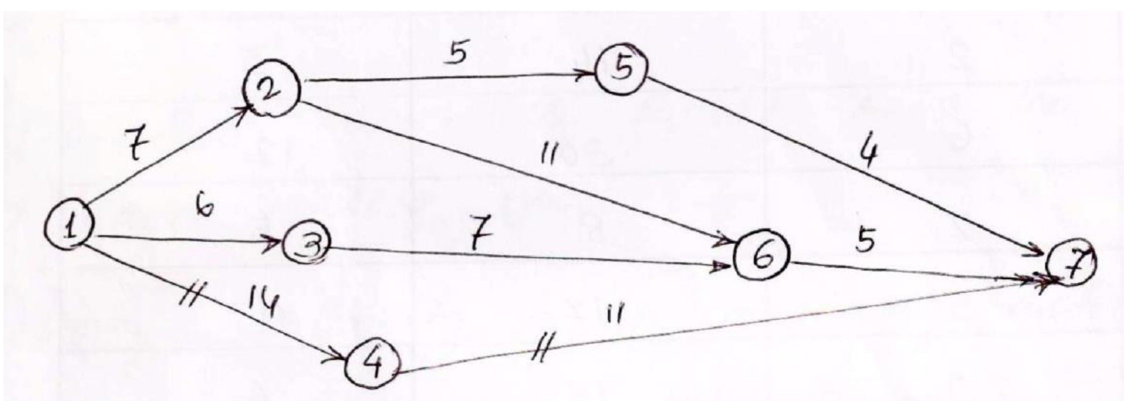
In order to find out the probability of completion in a given day, standard deviation (σ) for the critical path is to be determined using the formula

$$\sigma^2 = (\frac{tp - t0}{6})^2$$

So the required data are presented in the following

Activity	t_0	t_p	t_m	t_e	σ^2
1-2	3	15	6	7	4
1-3	2	14	5	6	4
1-4	6	30	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	6
3-6	3	15	6	7	4
4-7	3	27	9	11	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1

Using the expected time (te) value, the network is drawn as shown below.



a) 1-4-7 is critical path. The duration of project = 14+11= 25 days.

b) The sum of variances along the critical path = 16+16 = 32.

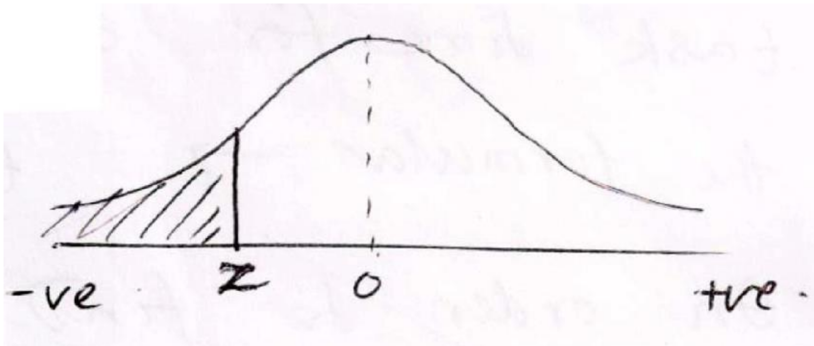
Now the standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{32} = 5.656$

Expected project duration, Te = 25 days,

Given D = 27 days,

With reference to Normal Distribution Curve

$$Z = \frac{D - Te}{\sigma} = \frac{27-25}{5.656} = 0.35$$



From Normal Distribution Table, for Z = 0.35, the fraction of (shadow) area = 0.637 i.e., 63.7%

So probability of completion of project in 27 days is 63.7%

[Note: If probability of completion of project will be given and the due date (D) will be asked to find out, then backward calculation can give the result].

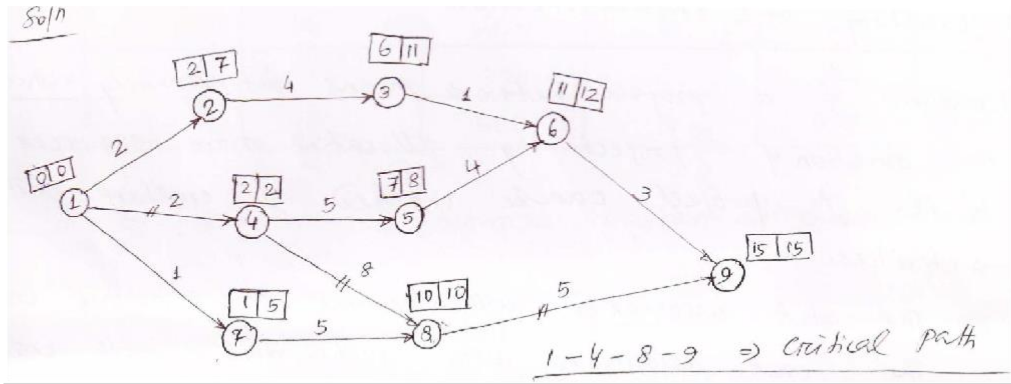
Problem-3:

A project schedule has the following characteristics →

Activity	1-2	1-4	1-7	2-3	3-6	4-5	4-8	5-6	6-9	7-8	8-9
Duration,day	2	2	1	4	1	5	8	4	3	5	5

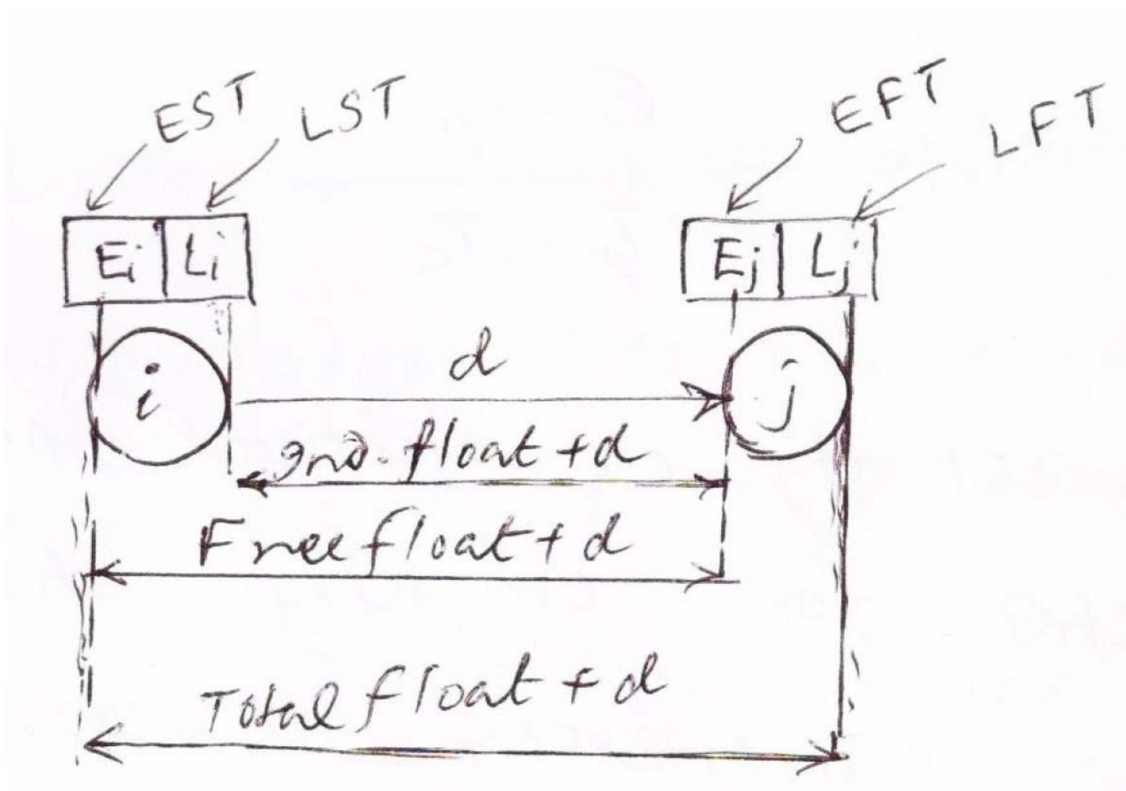
Construct the network and locate the critical path. Calculate the various time estimates and floats.

Solution:



The earliest start time (EST), the latest start time (LST) the earliest finish time (EFT) and latest finish time (LFT) are shown above at each node point by square block in the network diagram.

The various floats can be calculated w.r.t. the following figure :-



Activity	Duration	EST	LST	EFT	LFT	Total float	Free float	Independent float
1-2	2	0	0	2	7	5	0	0
1-4	2	0	0	2	2	0	0	0
1-7	1	0	0	1	5	4	0	0
2-3	4	2	7	6	11	5	0	-5
3-6	1	6	11	11	12	5	4	-1
4-5	5	2	3	7	8	1	0	-1
4-8	8	2	3	10	10	0	0	-1
5-6	4	7	8	11	12	1	0	-1
6-9	3	11	12	15	15	1	1	0
7-8	5	1	5	10	10	4	4	0
8-9	5	10	10	15	15	0	0	0

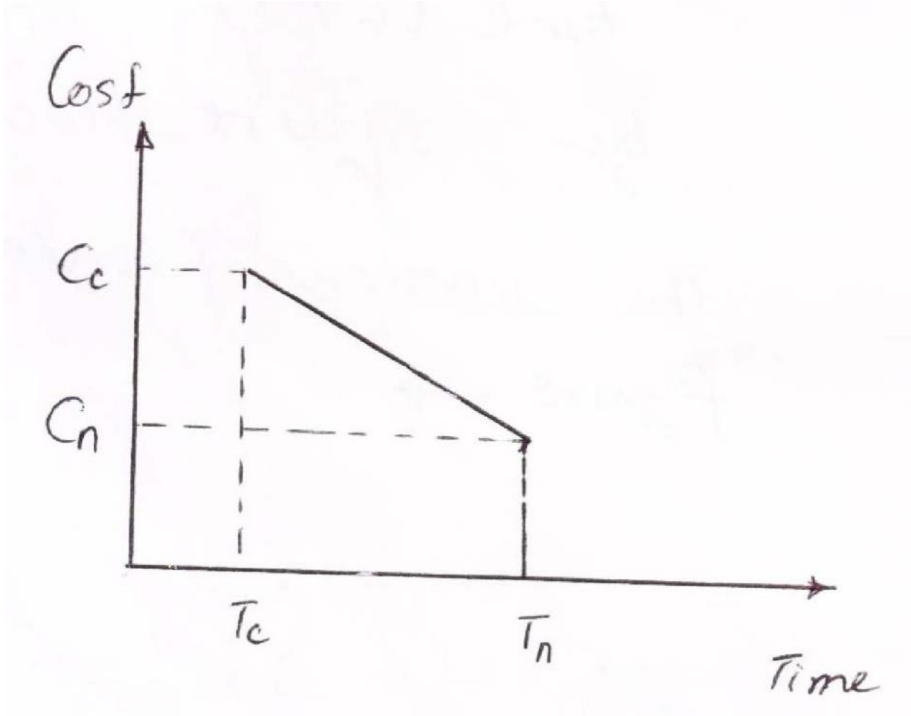
(C) Crashing of Project Network

Crashing of a project network means intentionally reducing the duration of project by allocating more resources to it. A project can be crashed by crashing critical activities.

The cost associated with normal time → normal cost

The cost associated with crashed time → crash cost.

It is obvious that the crash cost should be more than the normal cost.



The slope of crashing an activity is given by :

$$\text{Slope} = \frac{Cc - Cn}{Tn - Tc}$$

The activity on critical path selected for crashing should have minimum slope and for compression limit = min [crash limit, free float limit]

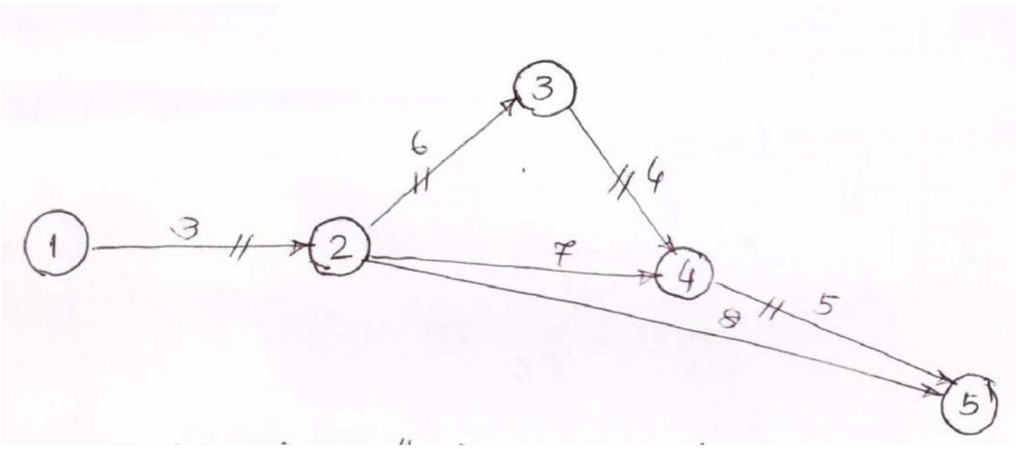
Typical Example on Crashing

For a network shown below, normal time, crash time, normal cost, crash cost are given in table. Contract the network by crashing it to optimum value and calculate optimum project cost. Indirect cost = Rs 100/- per day.

Activity	Normal		Crash	
	Time in days	Cost in Rs	Time in days	Cost in Rs
1-2	3	300	2	400
2-3	6	480	4	520
2-4	7	2100	5	2500
2-5	8	400	6	600
3-4	4	320	3	360
4-5	5	500	4	520

Solution:

By using the network diagram as shown below



The critical path → 1-2-3-4-5.

The project duration = 3+6+4+5+ = 18 days.

On this path, crash time = 2+4+3+4+ = 13 days

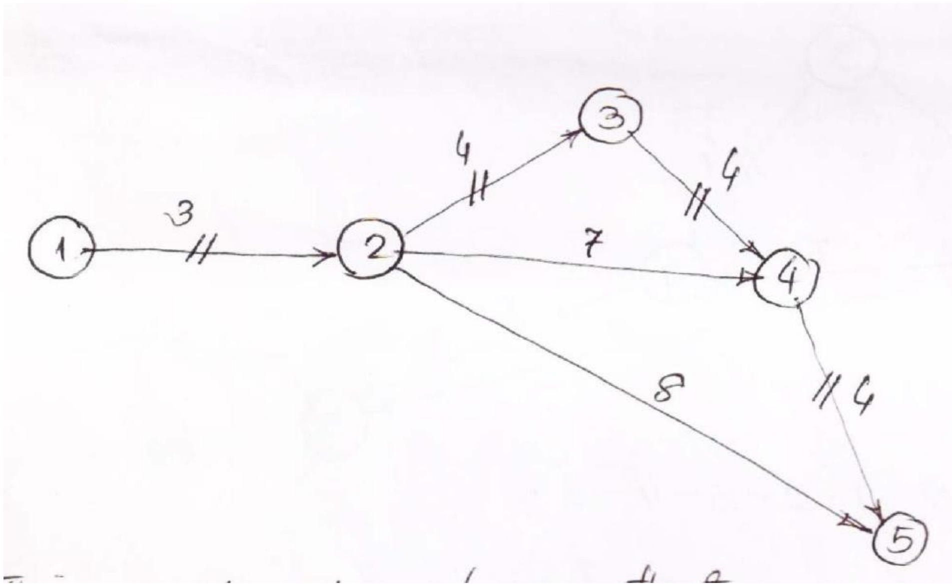
Normal cost = 300+480+2100+400+320+500 = Rs 4100/-

Crash cost = 400+520+2500+600+360+520 = Rs 4900

To contract the network in the 1st stage we should identify the activities on the critical path having lowest cost slope. For this purpose, the cost slopes are calculated as follows:

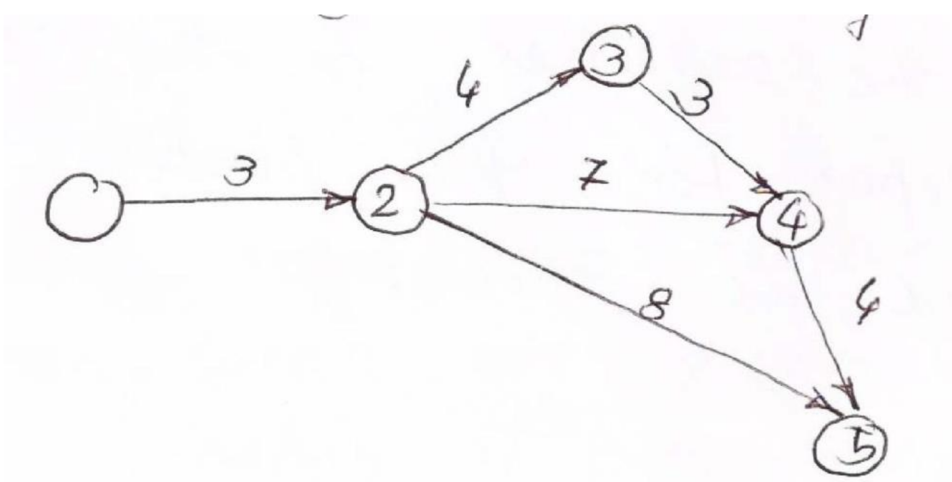
Activity	Normal		Crash		ΔC	ΔT	$\frac{\Delta C}{\Delta T}$
	Time, days	Cost, Rs	Time, days	Cost, Rs			
1-2	3	300	2	400	100	1	100
2-3	6	480	4	520	40	2	20
2-4	7	2100	5	2500	400	2	200
2-5	8	400	6	600	200	2	100
3-4	4	320	3	360	40	1	40
4-5	5	500	4	520	20	1	20

It is observed from the above table that, the critical path activities 2-3 & 4-5 have least cost slope. Therefore, these activities are 1st crashed. The modified network is drawn in the following.



This network shows that after crashing activity 2-3 by 2 days and activity 4-5 by 1 day, the critical path is same i.e. 1-2-3-4-5 and the project duration = 15days.

Now, in the 2nd stage, the least cost slope in the remaining activities is for activity 3-4 on the critical path. By crashing this activity, the new network diagram becomes:



Now 2 paths become critical path i.e. 1-2-4-5 and 1-2-3-4-5 and the duration = 14days.

Now we see that there is no other activity on both the critical paths which has cost slope less than indirect cost (i.e., Rs 100/-). This shows that this is the optimum network and hence optimum project duration = 14days.

Total direct project cost for this optimum condition = Direct cost for all activities on the network = Sum of costs of activities 1-2 , 2-3, 3-4, 4-5, 2-4, 2-5 = 300+520+360+520+2100+400 = Rs 4200/-

Indirect cost for 14 days = 14×100 = 1400/-

Total project cost after crashing = 4200+1400 = Rs 5600/-

Whereas, total cost with all normal activities = 4100+18×100 = Rs5900/-

So b crashing cost is reduced and time is also reduced.

(C) Project scheduling with limited resources

Usually the resources in project are:

- Manpower
- Equipments
- Money

These resources are limited. Hence the objective is to adjust noncritical activities between their EST & LFT such that the peak resource requirement is reduced. There are 2 types of problems under this category.

(1) Resource leveling (to minimize the peak requirement and smooth out period to period variation).
(2) Resource allocation (adjust the noncritical activities such that the resource requirement in each period is within the available range).

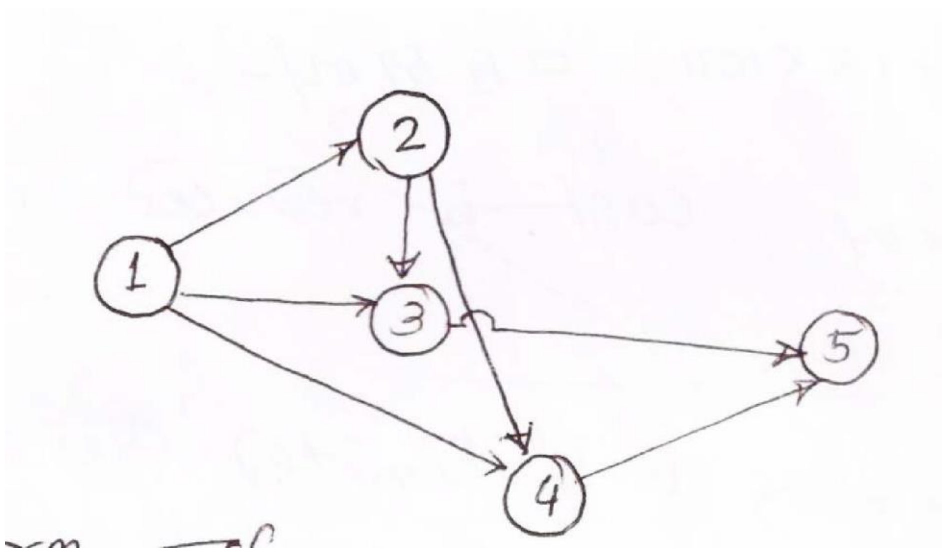
(1) Resource Leveling Technique

Ex: Consider the following problem of project scheduling to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

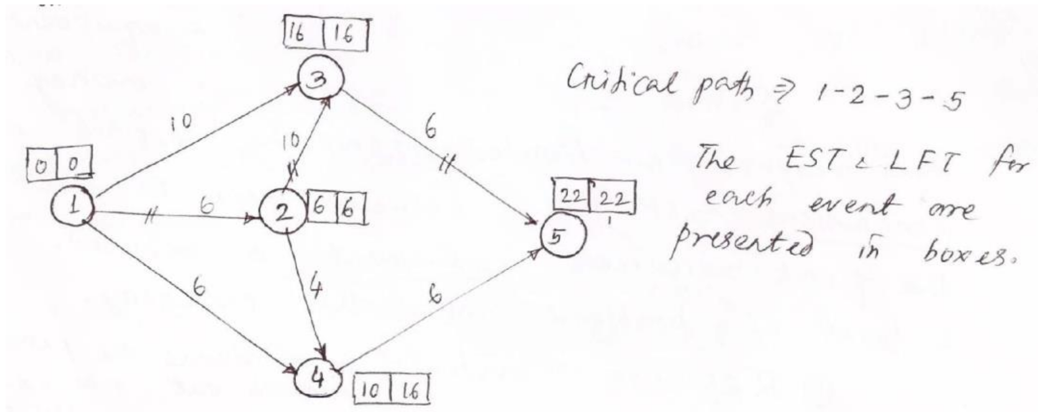
Activity	Duration	Manpower requirement
1-2	6	8
1-3	10	4
1-4	6	9
2-3	10	7
2-4	4	6
3-5	6	17
4-5	6	6

Solution:

The project network is shown below



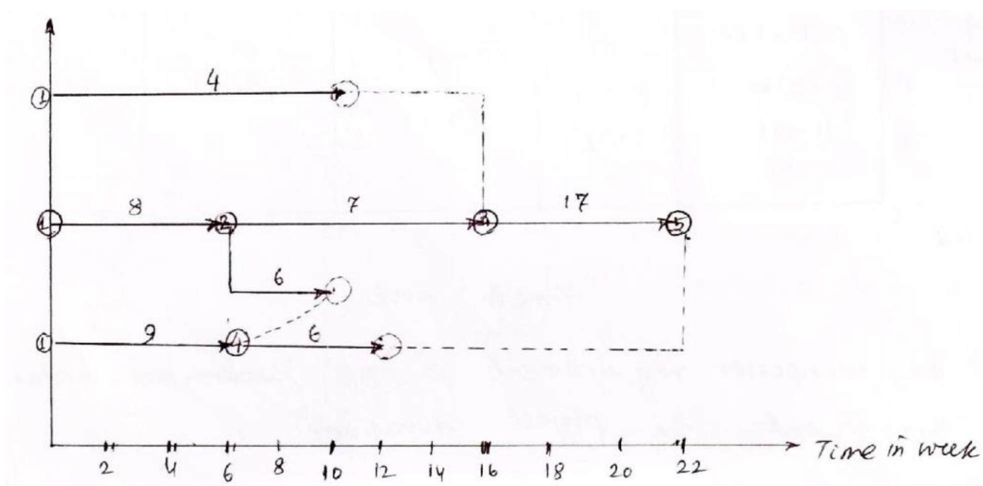
A better form of network will be obtained if event (3) will be above (2) and it is shown below.



Critical path = 1-2-3-5

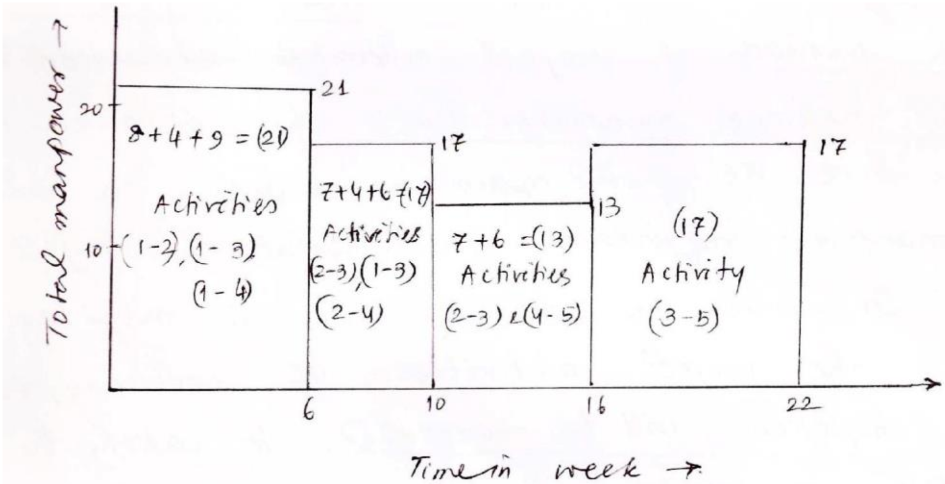
The EST & LFT for each event are presented in boxes.

The activities representation on a time state and corresponding manpower requirements are presented on the top of arrow as follows:

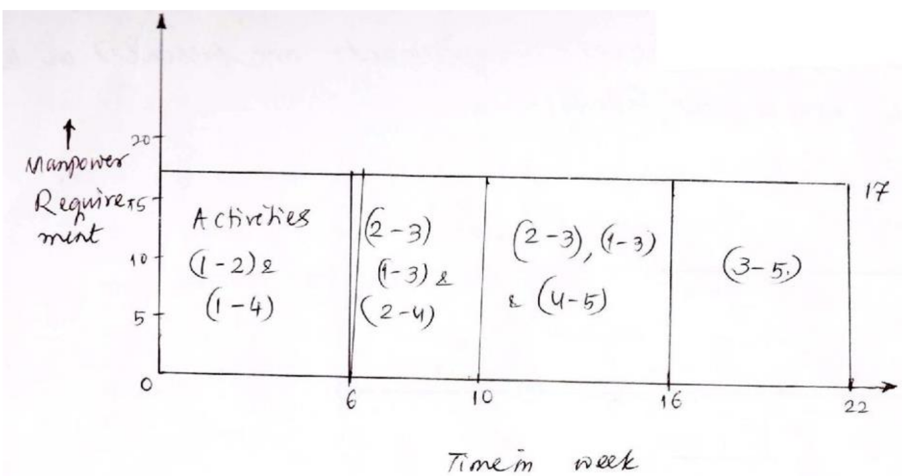


The corresponding manpower requirement histogram is as follows:

(shifting 4-5 towards right)



The peak manpower requirement is 21 and it occurs between 0-6 weeks. The activities which are scheduled during this period are: (1-2),(1-3) and(1-4). The activity 1-2 is a critical activity. So it should not be disturbed. Between activities (1-3) and(1-4), the activity (1-3) has a slack of 6 weeks. Hence it should be postponed to maximum extent (i.e., it can be started at the end of 6th week). The corresponding modification is shown in the following histogram.



The manpower requirement is now balanced/smothered throughout the project duration.

(2) Resource Allocation Technique

The objective of resource allocation is to reschedule the project activities so that the manpower requirement in each period of project execution is within the maximum manpower limit which is given as a constraints. Here we should aim to maintain a limit on the manpower requirement throughout the project duration. In order to achieve this, we may have to reschedule the project activities. If necessary, the project completion will be extended to satisfy the constraints on manpower limit.

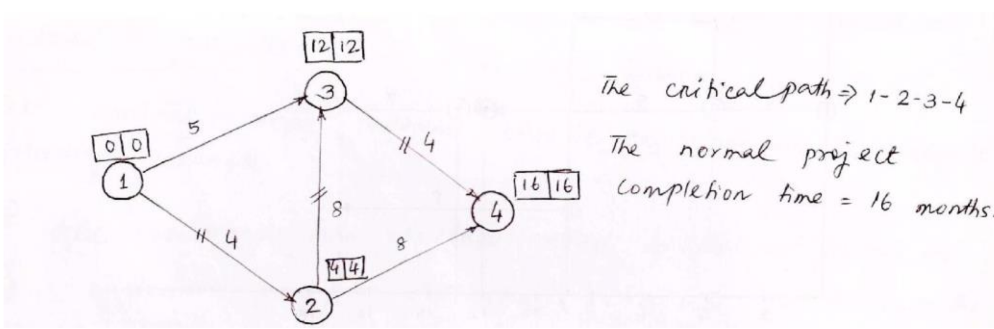
Ex. Consider the following problem

Activity	Duration in month	Manpower requirement
1-2	4	10
1-3	5	4
2-3	8	5
2-4	8	2
3-4	4	7

Reschedule the activities of the project with a maximum limit on the manpower requirement = 10

Solution:

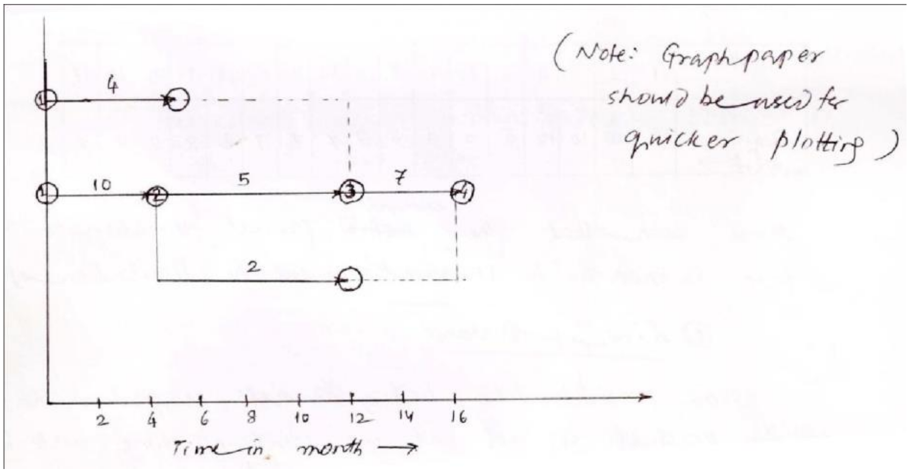
The project network and the various time values are shown below.



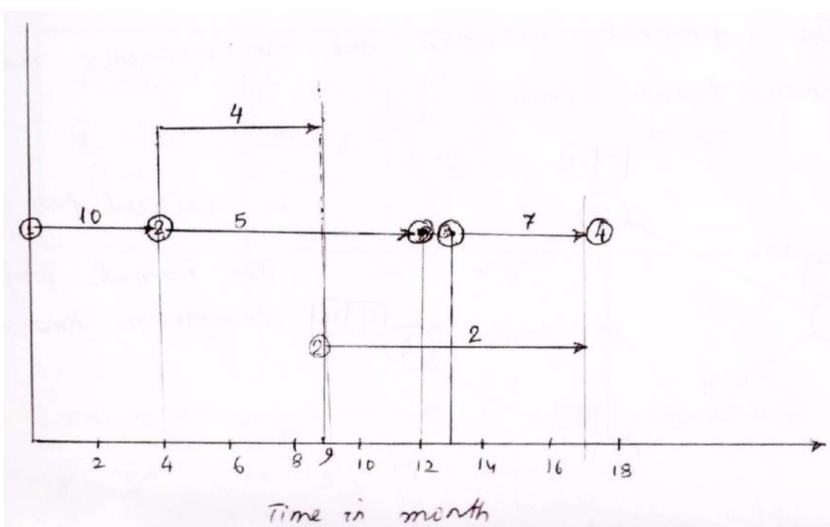
The critical path = 1-2-3-4

The normal project completion time = 16 months.

The normal project scheduling with manpower requirement (on the top of arrow) is shown below.



If the actual manpower allocated as per the project schedule is more than the upper limit of 1, then the non-critical activities are postponed with the most slack value so that the actual manpower on that month is less than or equal to the maximum limit. Inspite of this if the total manpower goes beyond the maximum limit, then the critical activity is to be postponed by some period such that total manpower is within the maximum limit. It is shown in the following.



Now the manpower requirement with month is presented in the following table.

Month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Manpower requirement	10	10	10	10	9	9	9	9	9	7	7	7	2	9	9	9	9

It is seen that the project duration is changed from 16 months to 17 months due to limitation of resource.

(C) Line of Balance

For some products like boiler, aircraft, computers, the delivery of the product is not at one point of time is spread over a time internal. For scheduling and control of these products, a graphic technique called Line of Balance (LOB) is quite essential.

For Line of Balance, the following information are required

i. Contracted schedule of delivery.
ii. Key operations in making the product (which need to be controlled).
iii. The sequence of key events.
iv. The expected/observed lead time with respect to delivery of final product.

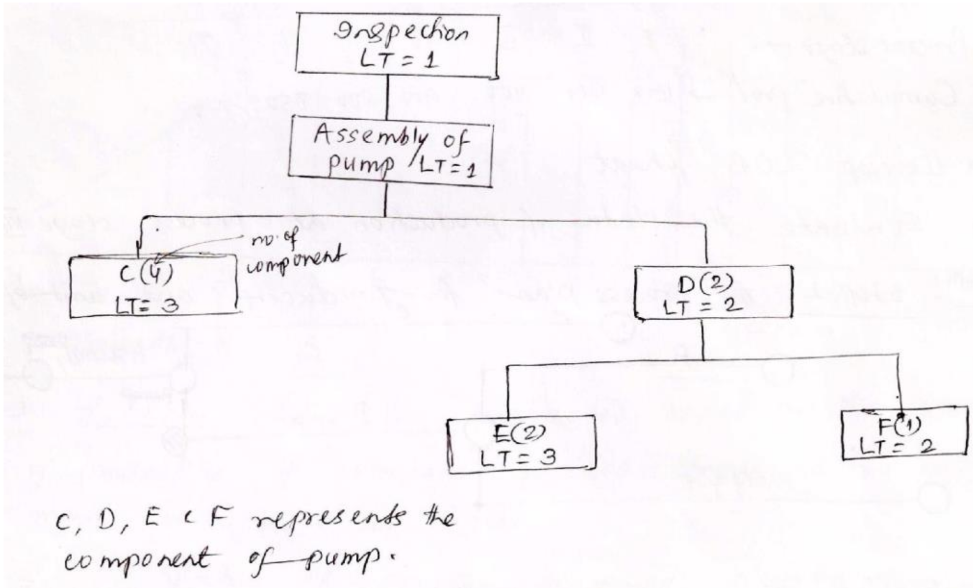
Based on the above information, a diagram is plotted which compress pictorially between the planned progress and actual progress → Line of Balance.

The various steps followed can be explained with the help of following example.

Ex: ABC company has received order to deliver pumps to its customers as per delivery schedule as shown in the following table.

Month	1	2	3	4	5	6
Quantity	100	200	200	200	300	400

The product structure explaining the subassemblies, inspection and test procedure along with lead time details is presents in the following figure.



The above information is presented in tabular form as presented in the following.

Sl. No.	Process stage	Components/subassembly	No. of components	Lead time in weeks
1	I	E	2	3
2	II	F	1	2
3	III	C	4	3
4	IV	D	2	2
5	V	Assembly of pump.	1	1
6	VI	Inspection	1	1
7	VII	Final product	1	-

The production schedule is for 6 months. After 4 months of production schedule, the cumulative number of units proceed at each process stage is shown below.

Cumulative production at the end of 4th month

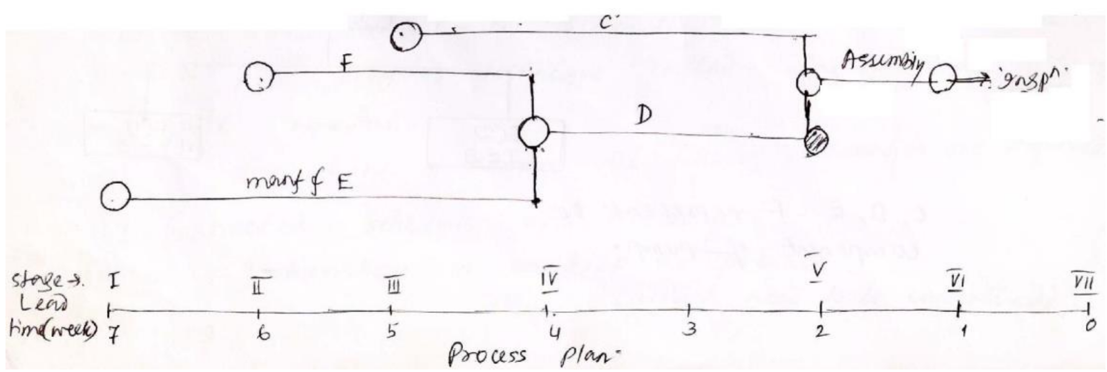
Process stage	I	II	III	IV	V	VI	VII
Cumulative production	1300	1200	1100	900	800	750	700

- Develop LOB chart
- Evaluate the status of production at process stage VII.

SOLUTION:

Step-1

The process plan for producing one unit of pump →



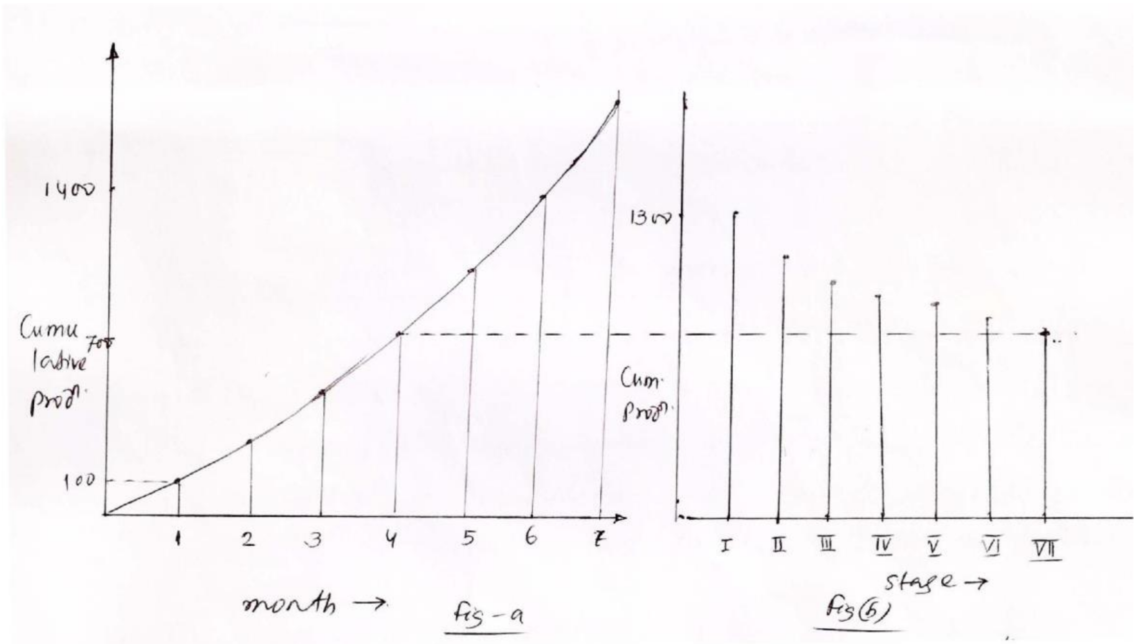
Step-2

The cumulative delivery schedule is computed as shown below.

Month	Quantity	Cumulative quantity
1	100	100
2	200	300
3	200	500
4	200	700
5	300	1000

6	400	1400

The cumulative delivery chart can be plotted as shown below.



Step-3

The LOB chart is constructed based on the cumulative no. of production of various process stages at the end of 4th month is shown in fig(b).

Status of process stage VII after 4th month, in order to determine this, a horizontal line is plotted corresponding to cumulative production volume at 4th month and producing this line on to the VII stage graph as shown in the previous page. If it touches the tip of cumulative production of stage VII, then it has perfect LOB.

CHAPTER-4: MODERN MANAGEMENT SYSTEM

© ISO 9000 series:

ISO stands for International Organization for Standardization. It is an international body consists of representatives from more than 90 countries . The national standard bodies of these countries are the members of this organization. This is a non-government organization. It provides common standards of goods and services on international trades.

ISO 9000 series has 5 nos. of international standards on quality management with different objectives as stated in the following.

ISO 9000 → Provides general guidelines for quality standard

ISO 9001 → Provide guidelines for supplier i.e.

- Design & development
- Production
- Installation
- Servicing

It has 20 elements

ISO 9002 → Provide guidelines for manufacturer i.e.

- Production
- Installation

It has 18 elements (i.e. excepts design & development as comp. to 9001)

ISO 9003 → Provides guidelines for test houses i.e.

- Final inspection
- Testing for laboratories and warehouses.

It has 12 elements

ISO 9004 → Provide guidelines for i.e.

- Quality management
- Quality assurance

Benefits of 9000 series

1. It helps to compete with global market.
2. Consistency in quality as ISO guidelines detect defective early so that rectification is possible.
3. Documentation of quality procedure adds clarify to quality system.
4. It gives guidelines and ensures adequate and regular quality training for all members of the organization.
5. It helps the customers to have cost effective purchase procedure.
6. The customers need not spend much time for quality testing for the firm holding ISO certificate.
7. The job satisfaction is more.
8. It also help in increasing productivity by reducing wastage and improvement of quality (for which revenue is high).

Steps in ISO 9000 Registration

The following steps are followed in ISO 9000 registration

1. Selection of appropriate standard i.e., ISO 9001/9002/9003 using guidelines given in ISO 9000.
2. Preparation of quality manual to cover all the elements in the selected standard.
3. Preparation of procedure and shop floor instruction which are used at the time of implementing the system. Also document these items.
4. Self auditing to check compliance for the selected standard/model.
5. Selection of a Registrar (an independent person with knowledge and experience to evaluate any one of the three quality system i.e., ISO9001/9002/9003) and application is to be submitted to obtain certificate for the selected quality standard/system/model.

The various elements involved in ISO 9001 & ISO 9002 are stated in the following.

Sl. No.	System requirement	ISO9001	ISO9002
1	Management responsibility	✓	✓
2	Quality system	✓	✓
3	Product identification & traceability	✓	✓
4	Inspection status	✓	✓
5	Inspection & testing	✓	✓
6	Inspection, measuring & test equipment	✓	✓
7	Controlling of non-conforming product	✓	✓
8	Handling, storage, packaging & delivery	✓	✓
9	Document control	✓	✓
10	Quality record	✓	✓
11	Training	✓	✓
12	Statistical technique	✓	✓
13	Internal auditing	✓	✓
14	Contract review	✓	✓
15	Purchasing	✓	✓
16	Process control	✓	✓
17	Purchaser supplied product	✓	✓
18	Corrective action	✓	✓
19	Design control	✓	×
20	Servicing	✓	✓

© <u>Poke a yoke</u>

Poke a Yoke is a Japanese terminology. In English it is called proofing. It implies mistake proofing why Poke a Yoke ?

1. To enable a person's mind free from maintaining watchful always which is practically impossible.
2. The operations can constructively do more value added activities.

Where Poke a Yoke helps ?

- Processing
- Assembly
- Inspection

- Packing/pashing/lebeling etc.

Basic types of Poke a Yoke

1. Prevention/Prediction type – Helps operator to recognize te defects before it occurs.
2. Detection type – Just immediately after the occurrence of defect/error.

Errors are inevitable but can be eliminated

Some of the Poke a Yoke devices help to avoid/eliminate defects are:

(1) Guide pins/Locator of different sizes
(2) Error detection buzzer & alarms
(3) Limit switches
(4) Logic switches etc.

(1) Guide pins/Locator pins of different sizes

The position of the arms of the bracket will be perfect by the help of guide pin as shown in fig. 4.1.

(2) Error detection buzzers and alarms

Alarm, buzzers, blinking lights are used to warm when any abnormality occurs in the system.

(3) Limit switch

It helps to control any excessive movement.

(4) Logic switch

Unless proper logic is maintained the switch will not operate. For example – Unless the cooling water circulation switch is made on, the power switch will not work.

Steps of Poke a oke

1. Select a troublesome area of interest.
2. Ask the workers to make a list of most common mistakes/troubles.

3. Workers should rank the mistakes according to their frequency of occurance, according to their importance and impact on the process.
4. Workers should develop Poke a Yoke devices in consultation with the engineer and design staff that eliminate the top ranked errors from both the lists (one list – all to frequency of occurance & other list – acc. to importance).
5. The implementation team (after proper analysis with regards to frequency and cost involved) adopt the proper Poke a Yoke devices.

Advantages of Poke a Yoke

1. Reduction of waste
2. Life of equipments / m/c_s are □
3. Quality products can be delivered to customers – Better customers satisfaction.
4. Improvement in employee relationship.

© Kaizen

Kaizen means change (Kai) to become good (zen). In otherwords, it means continuous improvement.

In fact, continuous improvement is required in all activities of the organization such as:

- Productivity improvement.
- New product development.
- Labour management relation.
- Total productive maintenance.
- Just in Time (JIT) production & delivery system. and
- Customer orientation etc.

The various activities of an organization where continuous improvement is required is presented under the Kaizen Umbrella as shown in fig. 4.2.

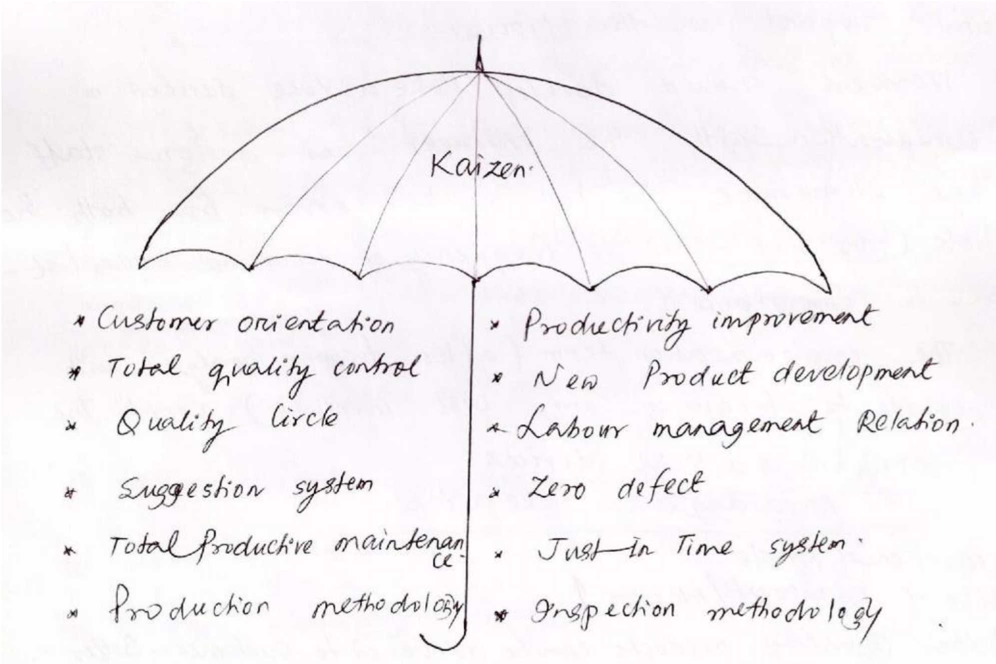


Fig 4.2: Kaizen umbrella

This continuous improvement in all areas are taken through small step by step process. Because various behavioral cultural and philosophical changes are better brought about through small step by step improvement than through radical changes.

Kaizen philosophy believes that people at all levels including the lowermost level in the organizational hierarchy can contribute to improvement. This is possible because, Kaizen asks for only small improvement.

Japanese and the Western (and by their influence our Indian) perception are presented in the following (fig 4.3 & fig 4.4) for comparison.

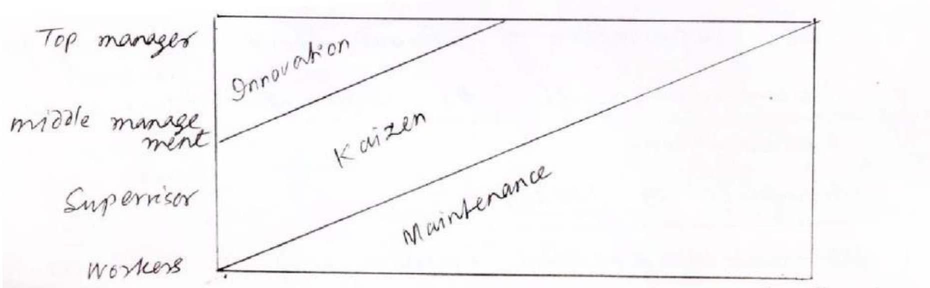


Fig 4.3: Japanese perception of job function.

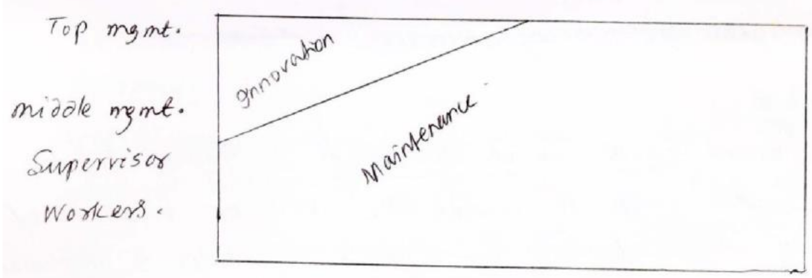


Fig 4.4: Western perception of job functions.

It is to be noted that Kaizen is to be performed at all levels from the top management to the lower level employees. Innovation should be supplemented by continuous improvement so that benefits of innovation keep on increasing over time instead of decreasing their utility due to constantly changing environment.

Steps for Implementation of Kaizen concept

1. Recognition of problem.
2. Writing the problem in the hand book.
3. Coming up with improvement ideas and writing the solution in the handbook.
4. Presentation of ideas and solutions to the supervisor/boss.
5. Implementation of the ideas.
6. The supervisor is informed about the daily progress.
7. Documentation of the implemented ideas on the Kaizen form.

Advantages of Kaizen

1) It involves creative ideas among the employees.
2) Involvement of workforce for continuous improvement.
3) Employees feel proud to work.

4) Employees get recognition.

© **Kanban**

Kanban mean card or visible record in Japan Toyota, Japan uses cards to control the flow of materials/inventory in a production process. It helps in implementing the JIT system in the industry. Althoug there are many different types of cards, the two main types used in Japan are :

(i) Production order card/production card. and
(ii) Withdrawal card/move card.

These 2 cards are used to indicate the amount and timing of material flow.

A move card or withdrawal card authorizes the transfer of one thousand container of a specified part from the work station (where the part is produced) to the station where it will be used (for example: machinery line assembly line).

A production card specifies the items and quality of production, the materials required, where to find them and where to store the finished items. The cards are attached to the container. Therefore, the number of containers used in a production process equals to the number of cards.

A card is marked with an identification number, a part number, a part description, a place of issue and the number of items in the standard container. Thus cards control the material flow.

The following figure (fig. 4.5) shows how the two cards are used to control the production flow. As a whole 8 steps are required as stated as following.

Step-1:

Accumulated withdrawal Kanban are taken to storage location A.

Step-2:

The production Kanban is detached (from the container where parts are present) and placed in the Kanban receiving post.

Step-3:

The parts on the container are checked against specification on the withdrawal Kanban and if satisfactory, withdrawal Kanban is attached to each containers having parts (with required specification). The continers are moved to the stocking location of the subsequent process.

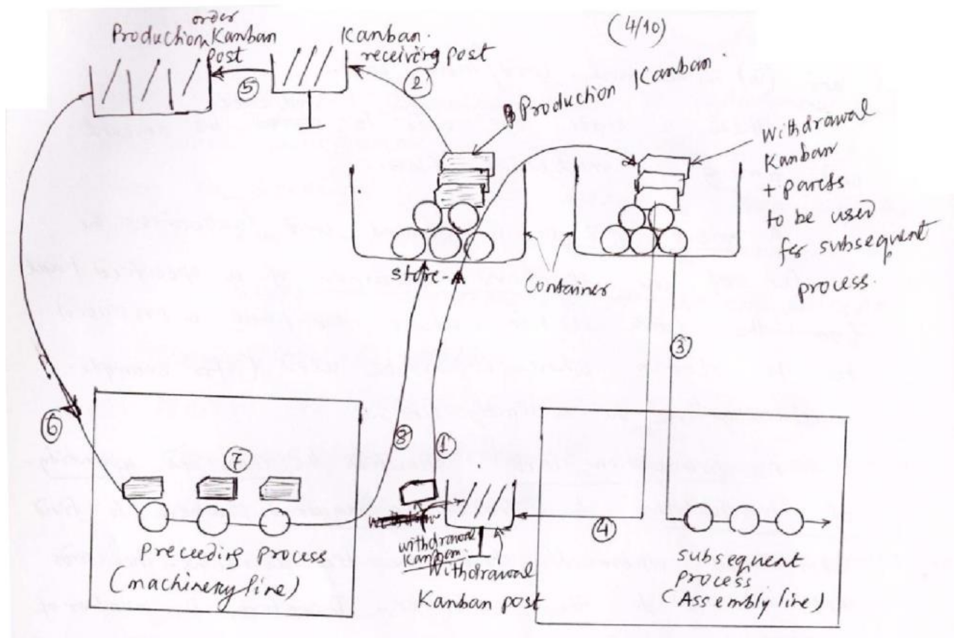


Fig 4.5: The flow of Kanban system card

Step-4:

When work begins on the container at the subsequent process, the withdrawal Kanban is detached and placed (or kept) on the post (i.e., withdrawal Kanban post).

Step-5:

The production sequence at the preceeding process begins with the removal of production cards from Kanban receiving post. These cards are reviewed and sorted before placing them in the production order Kanban post.

Steo-6:

The parts are produced in the sequence of the production order Kanban on the post.

Step-7:

The production order Kanban and the container move as a pair during processing.

Step-8:

In the last step, the finished units are transported to storage location A to support the production requirement of the subsequent process.

All work stations and suppliers are co-ordinated in a similar way to provide just in time quantities of materials.

General Operating Rules

The operating rules of the Kanban system are simple and are designed to facilitate the flow of materials while maintaining control in inventory level. These are as follows:

1. Each container must have a Kanban/card.
2. Materials are always obtained by subsequent process.
3. Container of parts must never be removed from a storage area without an authorizing Kanban.
4. All the containers should contain the same no. of good parts.
5. Defective parts should not be passed to the subsequent process.
6. Total production should not exceed the total amount authorized on the production order cards in the system. Similarly, the quantity of parts withdrawn for the subsequent process should not exceed the total amount authorized on the withdrawal cards in the system.

Determination of the no. of containers

The no. of containers needed to operate a work centre is a function of the demand rate, container size and the circulating time for a container. This can be obtained by using the formula :-

$$k = \frac{DT}{C}$$

Where, n = total no. of containers

D = demand rate (ex: x parts/day)

C = container size (i.e., no. of parts that can be held)

T = time for the container to complete the entire cycle (or lead time Ex. 0.1 day)

Conclusion:

Kanban system is a very simple and effective method of co-ordinating work centres and vendors. The organization must be well disciplined by this method.

© Quality circle

Quality circle may be defined as a small group of workers (5 to 10) who do the same/similar works voluntarily meeting together regularly during their normal working time, usually under the leadership of their own superior to

- Identity
- Analyse, and
- Solve work related problem.

The group present the solution to the management and whenever possible implement the solution themselves.

The QC concept was first originated in Japan in 1960. The basic cycle of a quality circle starts from identification of problem as shown in fig. 4.6

Philosophical basis of QC

1) It is a belief that people will take pride and interest in their work if they get autonomy and take part in the decision making process.

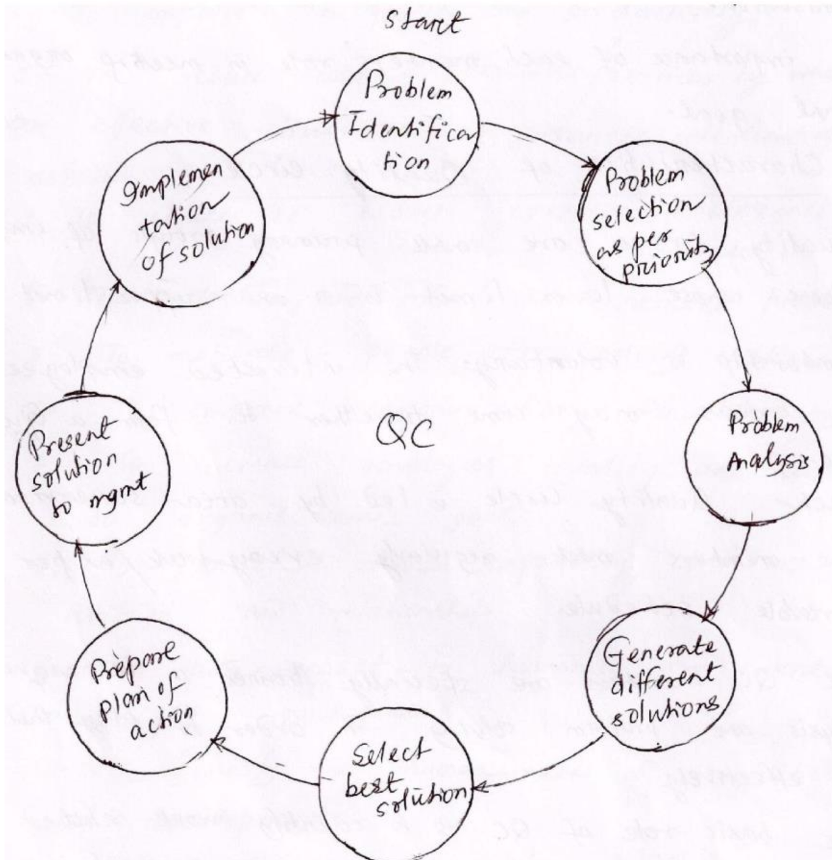


Fig 4.6 : Steps/ Actions of QC

2) It develops a sense of belongingness in the employees towards a particular organization.
3) It is also a belief that each employee desires to particulate in making the organization a better place.
4) It is a mean/method for the development of human resources through the process of training, work experience and particulate in problem solving.
5) A willingness to allow people to volunteer their time and effort for improvement of performance of organization.
6) The importance of each members role in meeting organizational goal.

Characteristics of Quality Circle

1) Quality circles are small primary groups of employees/workers whose lower limit is 3 and upper limit is 12.
2) Membership is voluntary. The interested employees in some areas may come together to form a quality circle.

3) Each quality circle is led by area supervisor.
4) The members meet regularly every week/as per aggreable schedule.
5) The QC members are specially trained in technique of analysis and problem solving in order to play their role effectively.
6) The basic role of QC is to identify work related problems for improving quality and productivity.
7) QC enable the members to exercise their hidden talents, creative skills etc.
8) It promotes the mutual development of their member through co-operative participation.
9) It gives job satisfaction because of identifying and solving challenging problems while performing job.
10) Members receives public recognition.
11) Members receives recognition from company's management in the form of memento, certificate etc.
12) As a result of above recognition there is development of self steem and self confidence of employees.

Objectives of QC

1) To improve the quality and productivity.
2) To reduce cost of products/services by waste reduction. Effective utilization of resources eliminating errors/defects.
3) To utilize the hidden creative intelligence of the employee.
4) To identify and solve work related problem.
5) To motivate people for solving challenging tasks.
6) To improve communication within the organization.
7) To increase employees loyality and commitment to organizational goal.
8) To enrich human capability, confidence, morale, attitude and relationship.
9) To pay respect to humanity and create a happy bright workplace.
10) To satisfy the human needs of recognition and self development.

© Just In Time (JIT)

The Just In Time production concept was first implemented in Japan around 1970's to eliminate waste of

- Materials
- Capital
- Manpower
- Inventory

Through out manufacturing system.

The JIT concept has the following objectives

- Receives raw materials just in time to be used.
- Produce part just in time to be used in subassemblies.
- Produce subassemblies just in time to be assembled into finished products.
- Produce and deliver finished products just in time to be sold.

In order to achieve these objectives, every point in the organization, where buffer stocks normally occur are identified. Then critical examination of the reason for such stocks are made.

A set of possible reasons for maintaining high stock is listed below.

- Unreliable/unpredicted deliveries.
- Poor qualities from suppliers.
- Increased varieties of materials.
- Machine break down.
- Labour obsentism.
- Frequent machine setting.
- Variation in operators capabilities.
- Schedule changes.
- Changing product modification.

In traditional manufacturing, the parts are made in batches, placed in the stock of finished product and used whenever necessary. This approach is known as "push system". Which means that parts are made according to schedule and are kept in inventory to be used as and when they are needed.

In contrast, Just In Time is a "pull system" which means that parts are produced in accordance with the order. It means the rate at which the products come out at the end of final assembly matches with the order quantity for that product. There no stock piles within the production process. This is also called zero inventory, stockless production, demand scheduling. Moreover, parts are inspected by the workers as they are manufactured. And this process of inspection takes a very short period. As a result of which workers can maintain continuous production control immediately identifying defective parts and reducing process variation. Therefore, the JIT system ensures quality products. Extra work involved in stockpiling parts are eliminated.

Advantages of JIT

Advantages of JIT are:

1. Exact delivery schedule is possible with JT practices.
2. Quality of product is improved.
3. Lower defect rate = lower inspection cost.
4. Lower – raw material inventory
    - Inprocess inventory
    - Finished product inventory

    Resulting lower product cost.
5. Satisfy customer without delay in delivery.
6. JIT helps in effective communication and reduce waste.
7. Less shop floor space in required.
8. Employees morale is high due to effective working environment.
9. JIT reduces scrap.
10. JIT reduces rework.

© TQM (Total Quality Management)

In order to make business excellency in the present scenario, it calls for continuously changing process/method/procedure that makes the goods/services delighting to customers. TQM is that management philosophy which create such organization. Total Quality Control (TQC) is only limited to manufacturing department.

Definition

TQM is an integrated organizational approach in delighting customers (both external and internal) by meeting their expectation on a continuous basis through every one involved with the organization working on continuous improvement basis in all products/processed along with proper problem solving methodology.

In other words, TQM means activities involving everyone (management persons & workers) in a totally integrated effort towards improving performance at every level. Improved performance means: Quality, cost, manpower development, quality of work life etc. It leads to increased customer and employee satisfaction.

In short, the definition implies – continuously meeting customers requirement at the lowest cost by utilizing the potential of all employees. Hence TQM can also be called as = Continuous Quality Improvement (CQI).

Why TQM? The reasons are:

1. Commitment to customers.
2. Improved productivity and quality.
3. Reduced cost.
4. Improved company's image.
5. Increased employees participation.

Principles of TQM

1. Agree with customers requirement.
2. Understand customers/suppliers.
3. Do the right things.
4. Do things right first time.
5. Various measures/action for success.
6. Continuous improvement is the goal.
7. Training is essential.
8. Better communication skills.

TQM should become a way of life. Once started, it is never ending. Continuous improvement is the goal. The top management should demonstrate their commitment to the subordinate consistently so that remaining members of the organization can follow it. Total means really total. TQM is a process of habitual improvement, where control is embedded within and is driven by the culture of organization.

Where applicable?

TQM is applicable to all functions/organization

TQM is traditional approach.

TQM approach	Traditional approach
1. No workers, no managers, only facilitators and team members	1. Blue collar employees – workers white collar employees – managers
2. Employees can voluntarily participate to look after various problems and take the pride of solving the problems.	2. Workers participation is legislated and take part at the time of bargaining (for money & other facilities).
3. Employee mostly focuses of organizational needs especially meeting the customers requirements.	3. Workers/managers focuses mostly on their needs.
4. Integrated co-operative (family like) working culture.	4. Conflict, win/loss style.
5. Open-ness, trust and respect is observed.	5. Secretive, distrust and hatred culture.
6. Here everyone is given importance.	6. Here only top managers (decision makers) are given importance.

TQM philosophy was implemented in every organization after 1970. From this time the boss & servant culture has been changed to friendly (family like) working environment.

MODULE-III CHAPTER-5: LINEAR PROGRAMMING

L.P. deals with optimization of object function subjected to certain constraints.

1. © Mathematical formulation of problem.

Mathematical formulation of a problem can be explained with the following example.

Ex: (1) A manufacturer produces 2 products (P_1 & P_2).

Each item of product P_1 requires - 4 hours of grinding

- 2 hours of polishing.

And each item of product P_2 requires – 2 hours of grinding

- 5 hours of polishing.

The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours per week. Profit per each item of product P_1 = Rs 3/- and Rs 4/- on product P_2. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the 2 types of product so that he may have maximum profit in a week.

Solution

Let the manufacturer plans to produce

X_1 from product P_1

& X_2 from product P_2 per week.

So the objective function = maximization of $Z = 3x_1 + 4x_2$

In order to produce these quantities,

(i) Total grinding hours needed/week = $4x_1+2x_2$.
(ii) Total polishing hours needed/week = $2x_1+5x_2$.

Maximum available grinding hours/week = 80 hours (40×2)

Maximum available polishing hours/week = 180 hours (60×3)

Therefore, the constraints =

$4x_1 + 2x_2 \leq 80$

And $2x_1 + 5x_2 \leq 180$

Also it is possible to produce –ve quantity so the mathematical formulation of the problem →

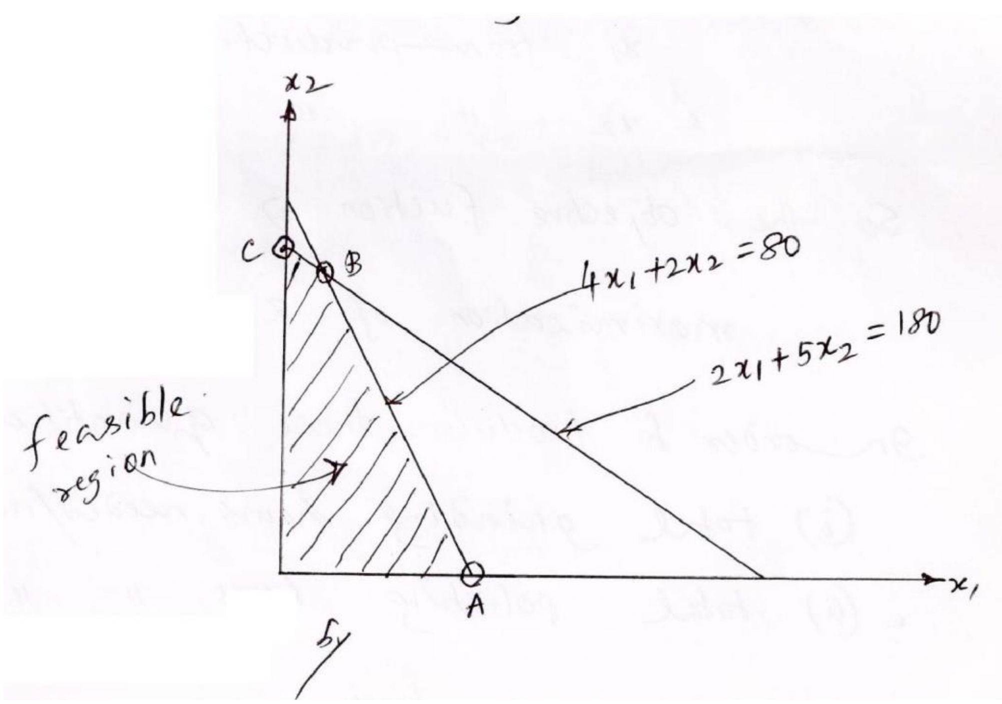
Find 2 real no.s (x_1 & x_2) such that $z = 3x_1 + 4x_2$ will be maximum subjected to constraints : (i) $4x_1 + 2x_2 \leq 80$

& (ii) $2x_1 + 5x_2 \leq 180$

$x_1, x_2 \geq 0$

© **Graphical solution method**

Graphical solution method is only applicable when there are only 2 variables. Considering the above problem, the graphical solution satisfying both the constraints, the various z values corresponding to point (A),(B) & (C) are:



$z/(A) = 3\times20+4\times0 = 60$

$z/(B) = 3x_1 + 4x_2$ where x_1 & x_2 are obtained by

solving $4x_1 + 2x_2 = 80$ ……. (1)

& $2x_1 + 5x_2 = 180$ …….(2)

Multiplying 2 in Eqn (2)

$\rightarrow \quad 4x_1 + 10x_2 = 360$

Subtracting eqn (1) $\rightarrow \quad \underline{4x_1 + 2x_2 = 80}$

$8x_2 = 280$

$\rightarrow x_2 = 35$

So, $4x_1 = 80 - 70 = 10 \rightarrow x_1 = 2.5$

So $z/(B) = 3 \times 2.5 + 4 \times 35$

$= 7.5 + 140 = 147.5$

$z/(i) = 3 \times 0 + 4 \times 36 = 144$

so, out of 60, 147.5 is highest.

So, the no. of products from $\underline{P_1 = x_1 = 2 \text{ nos}}$ (not (3) as it will be away from Feasible zone)

From $\underline{P_2 = x_2 = 35 \text{ nos.}}$

To get maximum profit and the value of maximum profit $= 3 \times 2 + 4 \times 35$

$= 146$ ans.

© **General Linear Programming problem**

When the no. of variables (x_1, x_2…….) is more than two, the LPP is called general LPP. For such LPP the graphical method fails & the only method used to solve is simplex algorithm.

Example of a General LPP (Diet Problem)

The nutrient contents of a number of different foodstuffs and the daily minimum requirement of each nutrient for a diet is given. Determine the balanced diet which satisfies the minimum daily requirement and at the same time has the minimum cost.

Solution

Mathematical formulation

Let there are n different types of foodstuffs available and m different types of nutrients required.

Let aij denote the number of units of nutrient I in one unit of foodstuff j where I = 1,2,3, ………… m

& j = 1,2,3,… …….. n

Then the total number of units of nutrient I in the desired diet is

$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots\ldots\ldots a_{in}x_n$

let bi be the number of units of the minimum daily requirement of nutrient i. Then, we must have

$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots\ldots\ldots a_{in}x_n \geq bi \quad i = 1,2,3, \ldots\ldots\ldots m$

also each x_j must be either positive or zero. Thus we also have xj ≥ 0

for j = 1,2,3,… ………….n

Final Considering of Cost, let c_j be the cost per unit of food j. Then the total cost of the diet in given by:

$Z=C_I X_I + C_2 X_2 + \text{---} + C_n X_n$

Thus the above problem of selecting the best diet reduces to the following mathematical form;

Find an n tuple (X_I, X_2, X_3 ------X_n) of real numbers such that;

(a) ai1 XI + ai2 x2 + --- ain xn ≥ bi   i= 1,2, - m
(b) xj1≥o   j=1 2 -- n

for which the expression (or objective function)

$z=cI\ xI + c2\ x2 \text{ --- } + cn\ xn$ may be minimum (least)

this is a general LLP.

It is general in the sense that the data bi, aij and cj are parameters, which for different sets of values will give rise to different problems. we can now give the formal definition of General LLP.

Definition General linear Programming Problem

Let z be a linear function on R" defined by:

(a) $z= c_I\ x_I + c_2\ x_2 + \text{---}\ c_n\ x_n$

where cjs are constants. Let aij be an mxn real matrix and let bI, b2 --- bm are set of constants such that

(b) $a_{II}\ x_I + a_{12}\ x_2 + \text{----} + a_{In}\ x_n \geq b_I$

$a_{2I}\ x_I + a_{22}\ x_2 + \text{----} + a_{2n}\ x_n \geq b_2$

$a_{mi}\ x_I + a_{m2}\ x_2 + \text{----} + a_{mn}\ x_n \geq b_m$

and (c) $x_j \geq o$ for j=1,2, --- n

the problem of determining an n tuple (x_I, x_2 --- x_n) which makes z a minimum (or maximum) and which satisfies (b) & (c) is called general liner programming problem.

Definition of objective function :

The linear function

$z= c_I\ x_I + c_2\ x_2 + \text{---}\ c_n\ x_n$

which is to be minimized (or maximized) is called the objective function of the general LPP.

Definition of solution to General LPP:

An n-type (x_I x_2 --- x_n) of real numbers which satisfies the constraints of a general LLP is called a solution to the general LPP.

Definition of feasible solution:

Any solution to a general LPP which also satisfies the non negative restriction of the problem is called feasible solution to the general LPP.

Definition of optimum solution

Any feasible solution which optimized (minims/maximizes) the objective function of a general LPP is called an optimum solution.

Definition of slack variable

Let the constraints of a General LPP be

$\sum_{j=I}^{n} a_{ij} x_j \leq b_i$ i= 1,2, --- k.

Then the nonnegative variables x_{n+I} which satisfy

$\sum_{j=I}^{n} a_{ij} x_j + x_{n+I} = b_i$

Are called slack variables

Definition of surplus variable

let the constraints of a general LPP be

$\sum_{j=I}^{n} a_{ij} x_j \geq b_i$ i= k+1, k+2, --- l.

then the nonnegative variable xn+I which satisfy

$\sum_{j=I}^{n} a_{ij} x_j - x_{n+i} = b_i$ i= k+1, k+2, --- l.

is called surplus variable

CHAPTER-6: SIMPLEX & DUAL SIMPLEX METHOD

1. © Simplex method

Introduction

Simplex method = Simplex technique = Simplex algorithm.

It is an iterative procedure for solving a linear programming problem in a finite no. of steps. This method provides an algorithm which consist in moving from one vertex of the region of feasible solution to another in such a way that the value of the objective function at the succeeding vertex is less (or more) than the preceding vertex so as to reach finally in the optimum solution.

The following important definitions are necessary to understand the simplex method.

(a) Basic solution

Given a system 'm' simultaneous linear equations in 'n' unknowns ($m<n$).

$Ax = b$,

Where A is a $m\times n$ matrix of rank m. Let B be any $m\times m$ submatrix formed by m linearly in dependent columns of A. Then a solution obtained by setting $n - m$ variable not associated with the columns of B equals to zero and solving the resulting system is called a basic solution to the given system of equations.

This can be explained with the following example.

Ex: Obtain basic solution to $x_1+2x_2+x_3 = 4$ --------------- (1)

$2x_1+x_2+5x_3 = 5$--------------- (2)

In this case $m = 2$ (i.e., Eq. (1) & Eq. (2))

$n = 3$ (i.e., x_1, x_2, x_3) $= m<n$

the above system of equation can be written as:

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

A x b

(2×3) ×(3×1) = (2×1)

The different sub matrix from matrix A are:

.$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$ & $\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}$

The basic solutions are obtained from the following equations:

(I) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ ------(1) setting $x_3 = 0$

(II) $\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix} \begin{bmatrix} x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ ---------- (2) setting $x_1 = 0$

(III) $\begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ ----------- (3) setting $x_2 = 0$

Considering Eq (1)

[$x_1+2x_2 = 4$] ×2

$\underline{2x_1+x_2 = 5}$

$4x_2-x_2 = 8 - 5 = 3$

$= 3x_2 = 3 = x_2 = 1$

$2x_1+1 = 5 = x_1 = 2$

So basic $x_1 = 2$, $x_2 = 1$ & non basic $x_3 = 0$

Similarly solving other 2 equations :

Basic $x_2 = 5/3$, $x_3 = 2/3$ non basic $x_1 = 0$

Basic $x_1 = 5$, $x_3 = -1$ non basic $x_2 = 0$

(b) Degenerate solution

A basic solution to the system is called Degenerate of one or more of the basic variables vanish.

Ex: Show that the following system of linear equations has a degenerate solution.

$2x1+x2-x3 = 2$

$3x1+2x2+x3 = 3$

Solution

The given system of linear equations can be written as:

$$\begin{pmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

A x b

The basic solution can be obtained as:

$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ----------- (1) by setting $x_3 = 0$

$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{bmatrix} x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ----------(2) by setting $x_1 = 0$

$\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} \begin{bmatrix} x1 \\ x3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ ----------(3) by setting $x_2 = 0$

Solving (1) Basic → $x_1 = 1, x_2 = 0$; non basic $x_3 = 0$

(2) Basic → $x_2 = 5/3, x_3 = -1/3$; non basic $x_1 = 0$

(3) Basic → $x_1 = 1, x_3 = 0$; non basic $x_2 = 0$

In each is 2 basic solutions at least one of the basic variables is zero. Hence two of the basic solutions are degenerate solutions.

(c) Basic feasible solutions

A feasible solution to a LPP which is also basic solution to the problem is called as basic feasible solution to LPP. Let the LPP be to determine x so as to maximize

$z = cx$

subjected to constraints:

$Ax = b$

& $x \geq 0$

Then x_B is a basic feasible solution to this problem if B is an m×m non-singular sub-matrix of A and $Bx_B = b$ $x_B \geq 0$

Looking to the solutions of earlier problem.

(1)
$x_2 = \frac{5}{3}$, $x_3 = -\frac{1}{3}$ (negative), $x_1 = 0$ ←

not basic feasible solu

(2) $x_1 = 1, x_2 = 0$ (basic); $x_3 = 0$ (non basic)

Basic feasible solution

(3) $x_1 = 1, x_3 = 0$(basic) ; $x_2 = 0$ (non basic)

(d) Associated cost vector

Let x_B be a basic feasible solution to the LPP:

Maximize $z = cx$

Subject to constraint $Ax = b$ and $x \geq 0$.

The vector $c_B = (c_{B1}, c_{B2} \ldots\ldots\ldots c_{Bm})$ where c_{Bi} are components of c associated with the basic variables is called the cost vector associated with the basic feasible solution x_B.

It is obvious that the value of the objective function for the basic feasible solution x_B, is given by:

$z_0 = c_B x_B$

(e) Improved Basic Feasible Solution

Let x_B and $\hat{x}_B$ be two basic feasible solution to the standard LPP, then $\hat{x}_B$ is said to be an improved basic feasible solution as compared to x_B if

$$\hat{c}_B \hat{x}_B \geq c_B x_B$$

Where $\hat{c}_B$ is constituted of cost components corresponding to $\hat{x}_B$.

(f) Optimum Basic Feasible Solution

A basic feasible solution xB to LPP –

Maximize $z = cx$

Subjected to $Ax = b$ and $x \geq 0$.

Is called an optimum basic feasible solution if

$$z_0 = c_B x_B \geq z^*$$

where z^* is value of objective function for any feasible solution.

© Fundamental Properties of solution

The fundamental properties of solution will allow us to arrive at the simplex algorithm. These properties are presented in the following in the form of theorems:

Theorem- 1: (Reduction of Feasible solution to basic feasible solution) If a LPP has a feasible solution, then it also has a basic feasible solution.

Theorem-2: (Replacement of a basic vector) Let a LPP have a basic feasible solution. If we drop one of the basic vectors and introduce a non-basic vector in the basic set, then the new solution obtained is also a basic feasible solution.

Theorem- 3: Improved basic feasible solution – stated earlier.

Theorem- 4 : <u>Unbounded solution</u>

Let there exist a basic feasible solution to a given LPP. If for at least one j, for which $y_{ij} \leq 0$ $(i = 1,2,3,\ldots\ldots m)$, $z_j - c_j$ is negative, then there does not exist any optimum solution to this LPP.

Theorem-5: condition of optimality

A sufficient condition for a basic feasible solution to a LPP to be an optimum (maximum) is that $z_j - c_j \geq 0$ for all j for which the column vector is not in the basis.

Theorem- 6: Any convex combination of k different optimum solutions to a LPP is again on optimum solution to the problem.

Theorem- 7: Minimax theorem

Let f be a linear function of n variables such that $f(x^*)$ is its minimum value for some point x^{T*}, x^T E R^n, Then $-f(x)$ attains its maximum at the point x^*. Moreover for x^T E R^n.

Minimum $f(x) = -\text{maximum}\{-f(x)\}$

© Computational procedure

The computational procedure for simplex algorithm can be explained with the help of a typical example.

Use simplex algorithm to solve the following LPP.

Maximize $z = 4x1+5x2+9x3$

Subjected to constraints : $x_1+x_2+2x_3 \leq 16$

$fx_1+5x_2+3x_3 \leq 25$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Solution:

Step-1: Check whether the objective function is to maximize or minimize. if it is a minimization case, then convert it into maximization by this relation minimize z = maximize $(-z)$.

Step-2: Check whether all bi are +ve. If any one bi is –ve, multiply the corresponding in equation by (-1).

Step-3: Convert all the inequations of constraints into equation by introducing slack/surplus variables

Now introducing slack variable, x_4 & x_5 the inequation of constraints because:

$X_1+x_2+2x_3+x_4 = 16$ --------------- (1)

$fx_1+5x_2+3x_3+x_5 = 25$ ---------------- (2)

and hence the new objected function becomes:

$z = 4x_1+5x_2+9x_3+0.x_4+0.x_5$ ----------------------(3)

step-4: Obtain an initial basic feasible solution to the problem in the form of

$x_B = {}^{-1}_{B}(b)$and put in the x_B column of the simplex table.

The set of equation (1) & (2) can be written as:

$$\begin{pmatrix} 1 & 1 & 2 & 1 & 0 \\ 7 & 5 & 3 & 0 & 1 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \end{bmatrix}$$

To obtain an initial basic feasible solution more easily the following matrix equation can be considered

i.e.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} x4 \\ x5 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \end{bmatrix}$$

:- x4 = 16

& x5 = 25

These values are substituted in the appropriate place in the initial simplex table.

Step-5: Compute the net evaluation i.e., $z_j - c_j$ ($j = 1,2,\ldots\ldots n$) by using the relation $z_j - c_j = c_B y_j - c_j$.

Examine the sign of $z_j - c_j$

If all the $z_j - c_j \geq 0$; than initial basic feasible solution x_B is an optimal basic feasible solution; otherwise if any one <0, we have to go to next step.

Now the initial simplex table becomes:

		c =	4	5	9	0	0	
c_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	
0	y_4	$x_4 = 16$	1	1	2	1	0	$\frac{16}{2} = 8$ ←
0	y_5	$x_5 = 25$	7	5	3	0	1	$\frac{25}{3} = 8\frac{1}{3}$
		$z = 0$	-4	-5	-9	0	0	$z_j - c_j$
					↑			

Step-6: If there are more than one –ve in $z_j - c_j$, choose the most –ve of them, let $Z_r - C_r$. If all $y_{ir} \leq 0$ ($i = 1,2,\ldots\ldots m$) then there is an unbounded solution to the given problem. If at least one $y_{ir} > 0$, then corresponding vector y_r enters the basis.

Step- 7: Compute the ratio $\frac{S_{Bi}}{y_{ir}}$, $y_{ir} > 0$, and choose the minimum of them. let the minimum be $\frac{S_{Bk}}{y_{kr}}$, then the vector y_k leaves the basis y_B. The common element is called pivotal element.

Step-8: Convert the pivotal element/leading element and all other elements in the column to zeros.

Step-9: Repeat the computational procedure until either an optimum solution is obtained or there is an indication of unbounded solution.

Following the procedure of simplex algorithm, the next simplex table becomes:

			$c =$ 4	5	9	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	
9	y_3	8	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{8}{\frac{1}{2}} = 16$
0	y_5	1	$\frac{11}{2}$	$7/2$	0	$-\frac{3}{2}$	1	$\frac{1}{7/2} = \frac{2}{7} \leftarrow$
		$z = 72$	$\frac{1}{2}$	$-\frac{1}{2}$ $\uparrow$	0	$\frac{9}{2}$	0	

The next simplex table will be :-

			$c =$ 4	5	9	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
9	y_3	$\frac{55}{7}$	$-\frac{2}{7}$	0	1	$5/7$	$-\frac{1}{7}$
5	y_2	$2/7$	$\frac{11}{7}$	1	0	$\frac{-3}{7}$	$\frac{2}{7}$
		$72\frac{1}{7}$	$\frac{9}{7}$	0	0	$\frac{30}{7}$	$\frac{1}{7}$

Since all zj – cj are ≥ 0 so no repeat procedure is required and the maximum value of z = $72\frac{1}{7}$ ans

© Concept of duality in simplex methd

The original LPP is called → primal problem

This original LPP can also be expressed in different form – called → Dual problem.

There is an unique dual problem associated with the primal problem and vice versa.

The following example will clearly explain the duality of original.

Ex: The amount of vitamins (v_1 & v_2) present I 2 different food (f_1 & f_2), cost and daily requirement are presented in the following table.

Vitamin	Food		Minimum daily requirement
	F1	F2	
V1	2	4	40
V2	3	2	50
Cost	3	2.5	

The problem is to determine the minimum daily requirement of the 2 foods to have minimum cost.

Solution:

In order to formulate the problem mathematically, let x_1 & x_2 are the units of foods from f_1 & f_2 respectively. So the objective function is to.

Minimize $z = 3x_1+2.5x_2$

Subject to constraints:

(i) $2x_1 +4x_2 \geq 40$

(ii) $3x_1 +2x_2 \geq 5.$

Formation of dual problem from this primal problem.

Suppose there is a wholesale dealer who sales 2 vitamins v_1 & v_2 along with other commodities. The dealer knows very well that the foods f_1 & f_2 have market values only because of their vitamin content.

Now the problem of the dealer is to fix maximum per unit selling prices for the 2 vitamins in such a way that the resulting prices of foods f_1 & f_2 does not exceed their existing market price.

Let the dealer decide to fix up 2 prices at w_1 & w_2 per unit respectively. Then the dealer problem can be stated mathematically.

Maximize $z = 40w_1 + 50w_2$

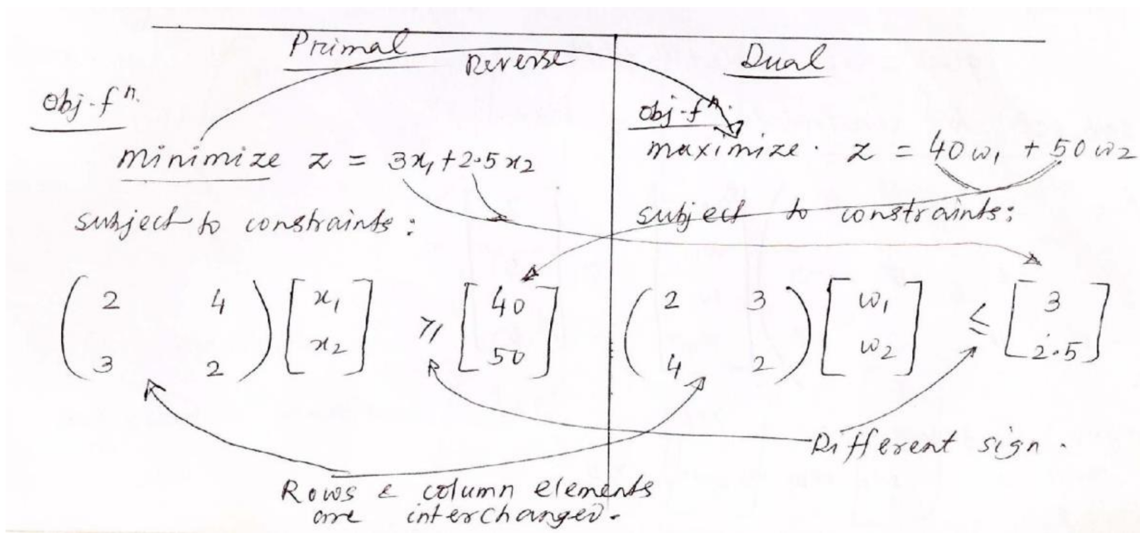
Subject to constraints:

$2w_1 + 3w_2 \leq 3$

$4w_1 + 2w_2 \leq 2.5$

$w_1\ w_2 \geq 0$

A comparison between primal & dual problem is presented as follows:



Ex: Obtain the dual problem of the following LPP.

Maximize $f(x)\ 2x_1+5x_2+6x_3$

Subject to constraints

$5x_1+6x_2-x_3 \leq 3$

$-2x_1+x_2+4x_3 \leq 4$

$X_1-5x_2+3x_3 \leq 1$

$-3x_1-3x_2+7x_3 \leq 6$

$x_1\ x_2\ x_3 \geq 0.$

Also verify that dual of the dual problem is the primal problem.

Solution: The primal can be restated as : Determine x_1 x_2 x_3 so as to maximize :-
$f(x) = 2x_1+5x_2+bx_3$.

Subjected to constraints:

$$\begin{pmatrix} 5 & 6 & -1 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \leq \begin{bmatrix} 3 \\ 4 \\ 1 \\ 6 \end{bmatrix}$$

<u>4*3</u> <u>3*1</u> <u>4*1</u>

Thus if $w = (w_1,w_2,w_3,w_4)$ are the dual variable then the problem is to determine w so, as to minimize.

$d(w) = 3w_1+4w_2+w_3+6w_4$

<u>subject to constraints:</u>

$$\begin{pmatrix} 5 & -2 & 1 & -3 \\ 6 & 1 & -5 & -3 \\ -1 & 4 & 3 & 7 \end{pmatrix} \begin{bmatrix} w1 \\ w2 \\ w3 \\ w4 \end{bmatrix} \geq \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

3*4 4*1 3*1

$w1\ w2\ w3\ w4 \geq 0$

i.e. minimize

$g(w) = 3w_1+4w_2+w_3+6w_4$

subjected to constraints:

$5w_1-2w_2+w_3 - 3w_4 \geq 2$

$6w_1+w_2-5w_3-3w_4 \geq 5$

$-w_1+4w_2+3w_3+7w_4 \geq 6$

$w_1,w_2,w_3,w_4 \geq 0.$

Now, we can restate this dual problem as:

Maximize $h(w) = -3w_1 - 4w_2 - w_3 - 6w_4$

Subjected to constraints

$$(-1)\begin{pmatrix} 5 & -2 & 1 & -3 \\ 6 & 1 & -5 & -3 \\ -1 & 4 & 3 & 7 \end{pmatrix}\begin{bmatrix} w1 \\ w2 \\ w3 \\ w4 \end{bmatrix} \le \begin{bmatrix} -2 \\ -5 \\ -6 \end{bmatrix}$$

$w_1, w_2, w_3, w_4 \ge 0$

We can write dual of this dual problem as

To minimize

$h(x) = -2x_1 - 5x_2 - 6x_3$

subjected to constraints:

$$(-1)\begin{pmatrix} 5 & 6 & -1 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{pmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \ge \begin{bmatrix} -3 \\ -4 \\ -1 \\ -6 \end{bmatrix}$$

$x1, x2, x3 \ge 0$

i.e, maximize $f(x) = 2x1 + 5x2 + 6x3$

subjected to constraints:

$$.\begin{pmatrix} 5 & 6 & 1 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{pmatrix}\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} \le \begin{bmatrix} 3 \\ 4 \\ 1 \\ 6 \end{bmatrix} \quad \leftarrow \text{original form proceed}$$

© **Dual Simplex Algorithm**

The various steps of dual simplex algorithm in presented in the following.

Step-1: Convert the minimization L.P.P. into maximization if it is in the minimization form.

Converts into ≥ type inequations representing the constraints into ≤ type by multiplying with (-1).

Step-2: Introduce slack variables in the constraints and obtain an initial basic solution. Put this solution in the starting dual simplex table.

Step-3: Test the nature of $z_j - c_j$ in the starting simplex table.

(a) If all $z_j - c_j$ and x_{Bi} are non-negative for all i & j then an optimum basic feasible solution has been obtained.
(b) If all $z_j - c_j$ are non- negative and at least one basic variable say x_{Bi} is negative, then go to step 4.
(c) If at least one z_j-c_j is negative, the method is not applicable to the given problem.

Step-4: Select the most negative x_{Bi}s. The corresponding basic vector then leaves the basis set y_B. Let x_{Bk} be the most negative basic variable so that y_k leaves y_B.

Step-5: Test the nature of y_{ki}, j = 1,2,………n.

(a) If all y_{ki} are non-negative, there does not exist any easible solution to the given problem.
(b) If at least one y_{ki} is negative, compute the replacement ratios $\{\frac{Zj-cj}{yki}, y_{ki} < 0\}$ j = 1,2,…….n. and choose the maximum of these. The corresponding column vectors, say y_r (corresponding to $\frac{Zr-cr}{ykr}$), then enters the basic set y_B.

Step-6: Test the new iterated dual simplex table for optimality.

Repeat the procedure until either an optimum feasible solution has been obtained (in finite number of steps) or there is an indication of the non-existence of a feasible solution.

<u>Sample problem</u>

Use dual simplex method to solve the LPP

Minimize $z = x_1+2x_2+3x_3$

Subject to constraints:

$X_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

solution:

the objective function should be maximize z = -x1-2x2-3x3

by introducing slack variables x4, x5, x6 ≥0 in the constraints (after converting into ≤ condition) the equations for constraints would be

$-x_1 + x_2 - x_3 + x_4 = -4$

$X_1 + x_2 + 2x_3 + x_5 = 8$

$0 \times x_1 - x_2 + x_3 + x_6 = -2$

$$:-\begin{pmatrix} -1 & +1 & -1 & +1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -1 & +1 & 0 & 0 & +1 \end{pmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ -2 \end{bmatrix}$$

Considering x1,x2&x3 = 0 the initial basic solution would be obtained from.

$$.\begin{pmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & +1 \end{pmatrix} \begin{bmatrix} x4 \\ x5 \\ x6 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \\ -2 \end{bmatrix}$$

i.e.,

$x_4 = -4,$

$x_5 = 8$

$x_6 = -2$

now the starting dual simplex table:-

			-1	-2	-3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	-4	-1^*	1	-1	1	0	0
0	y_5	8	1	1	2	0	1	0
0	y_6	-2	0	-1	1	0	0	1
		0	1	2	3	0	0	0

First iteration: y_4 is dropped and y_1 is introduced. Hence the resulting dual simplex table becomes:

			-1	-2	-3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-1	y_1	4	1	-1	1	-1	0	0
0	y_5	4	0	2	1	1	1	0
0	y_6	-2	0	-1^*	1	0	0	1
		-4	0	3	2	1	0	0

Second iteration: y_6 is dropped and y_2 is introduced and the resulting table:-

			-1	-2	-3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-1	y_1	6	1	0	0	-1	0	-1
0	y_5	0	0	0	3	1	1	2
-2	y_2	2	0	1	-1	0	0	-1
		-10	0	0	5	1	0	3

Now since all $x_j-c_j \geq 0$ and also all $x_{Bi} \geq 0$ an optimum basic feasible solution has been reached.

Hence the optimum basic feasible solution to the given LPP is $x_1=6$, $x_2=2$, $x_3=0$ and minimum $z = -(-10) = 10$ ans

MODULE- IV

CHAPTER-7: TRANSPORTATION PROBLEM

1. Introduction

The transportation problem is one of the subclass of LPPs in which the objective is to transport various quantities of a single homogeneous commodity (that are initially stored at various origins) to different destinations in such a way that the total transportation cost should be minimum.

Ex:

(1) A shoe company which has in diff. manufacturing units located at different places of India. The total products are to be transported to 'n' retail shops in 'n' different places of India.

(2) Similar case for cold drink company.

A transportation table is used to solve this transportation problem as shown below (like simplex table to solve LPP)

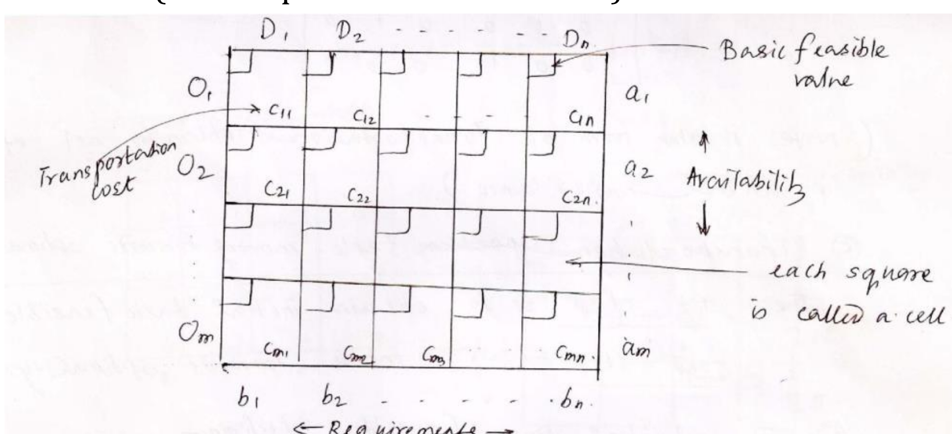


Fig 7.1: Transportation Table (m origin & n destination. Here m = 4, n = 5).

The various origins capacity and destination requirements are presented in fig. 7.1

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem – $b_I + b_2 + b_3 \text{ --- } b_n = a_I + a_2 + a_3 \text{ --- } a_m$. i.e. total requirements = total availability.

Matrix form of transportation problem

Minimize $z=cx \qquad c, x^T \in R^{mn}$

Subject to constraints

$Ax = b \qquad x \geq o \; b^T \in R^{mn}$

Where $x= [x_{II} \ldots\ldots x_{In}, x_{21} \ldots. x_{2n} \ldots. x_{mI} \ldots x_{mn}]$

$b= [a_I, a_2 --- a_m, b_I --- b_n]$, A is an (m+n) xmn real matrix containing the coefficient of the constraints and is the cost vector.

It should be noted that the elements of A are either O or +I. For a transportation problem involving 2 origin & 3 estimations (m=2, n=3) the matrix A is given by

$$A = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ \hdashline 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

(Note: matrix form of transportation problem is not of practical importance).

Transportation table moving towards optimality The 1st step is to obtain initial basic feasible sol^n. With 2nd step " " move towards optimality.

The initial basic feasible solution

The various methods used to obtain the initial basic feasible are

(i) North west corner Rule
(ii) Row minima method
(iii) Column minima method
(iv) Matrix minima method
(v) Vogels approximation method

(i) North west Corner rule

In this method the first assignment is made is the cell occupying the upper left hard (north-west) corner of the transportation table. a sample problem will clarity this method/rule.

Determine the initial basic feasible solution to the following transportation problem.

	D1	D2	D3	D4	
01	6	4	1	5	14
02	8	9	2	7	16
03	4	3	6	2	5
	6	10	15	4	

Requirements

Soln – Feasible solution exist if sum of availability = sum of requirements ie, 14+16+5 = 35 = 6+10+15+4

The transportation Table-

6	4	1	5	14
8	9	2	7	16
4	3	6	2	5
6	10	15	4	

Applying north-west corner rule →

6 →	8 ↓			14
	2 →	14 ↓		16
		1 →	4	5
6	10	15	4	

It has been proved theoretically that a feasible solution obtained by North-west corner rule (or by other method) is a basic solution.

(ii) Row minima method (with example)

Determine the initial basic feasible solution to the T.P. Using row minima method.

	A	B	C	Avai.
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Req.	4	2	2	

Soln.

50	1 30	220
90	45	170
250	200	50

	1		
90	1 45	170	

→

	1		
2	1		2
250	200	50	
4		2	

	1		
2	1		
250	200	2 50	4
2		2	

→

	1		
2	1		
2		2	2
2			

Now Transportation cast = 1x30+2x80+1x45+2x250+2x50 = 855 unit cost/ Rs.

(iii) Column minima method

It is similar to row minima method.

(iv) Matrix minima method

Obtain an initial basic feasible solution using matrix minima method.

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	24

Solution:

1	2	3	4	6
4	3	2	6 0	8
4 0	2	2	1	10
4	6	8	6	

→

	6 2	3		6
	3	2	6	2
4	6 2	2		6
	6	8		

	6			
		2	6	2
4	0	6		6
		8		

→

	6		
		2	6
4	0	6	

The transportation cost (TL) = 2x6+2x2+6x0+4x0+0x2+6x2

= 12+4+12 = 28 (Ans)

(v) Vogels Approximation method

Ex.: Obtain initial basic feasible solution using vogel's approximation method.

5	1	3	3	34
3	3	5	4	15
6	4	4	3	12
4	-1	4	2	19
21	25	17	17	80

Solution:

5	1	3	3	34(2)
3	3	5	4	15(0)
6	4	4	3	12(1)
4	19 -1	4	2	19(3)
21 (2)	25 (2)	17 (1)	17 (0)	

→

5	6 1	3	3	34(2)
3	3	5	4	15(0)
6	4	4	3	12(1)
	19			
21 (2)	6 (2)	17 (1)	17 (0)	

5	6	3	3	28(0)
15 3		5	4	15(1)
6		4	3	12(1)
	19			
21 (2)		17 (1)	17 (0)	

←

→

5	6	3	3	28(0)
15				
6		12 4	3	12(1)
	19			
6 (1)		17 (1)	17 (0)	

6 5	6 1	5 3	17 3	28
15 3	3	5	4	
6	4	12 4	3	
4	19 -1	4	2	
6		5	17	

Total cost = $6\times5+6\times1+5\times3+17\times3+15\times3+12\times4+19\times(-1)$

$=30+6+15+51+45+48-19 = 176.$

With the above techniques the initial basic feasible solution is obtained.

(B) Moving towards optimality

The procedure can be explained by taking the following example.

	D1	D2	D3	D4	Avai.
01	6	4	1	5	14
02	8	9	2	7	16
03	4	3	6	2	5
Req.	6	10	15	4	

Step -1

The initial basic feasible solution is first obtained by any of the 5 methods. let by following the North-west corner rule the initial basic feasible sop –

6 6	8 4	1	5
8	2 9	14 2	7
4	3	1 6	4 2

Step – 2

The dual variables ui & vj are first calculated for all basic cells such that

ui+vj = aj for all basic cells. These are calculated by assuming u1=o as presented in the following.

$u_I+v_I= 6 \Rightarrow o+v_I = 6 \Rightarrow v_I = 6$

$u_I+v_2= 4 \Rightarrow o+v_2 = 4 \Rightarrow v_2 = 4$ and so or.

6 6	8 4	1	5	0
8	2 9	14 2	7	5
4	3	1 6	4 2	9
6	4	-3	-7	

Step – 3

Now compute the net evaluation zij for all non basic cells. i.e., zij-cij = vi+vj – cij for all nonbasic cells, and present in the non basic cell at the right top corner with paren these (or bracket), if all the values of net evaluation is –ve, then this is the optimal solution. Otherwise go to step -4. Following this rule the net evaluations for all and shown aside.

6 6	8 4	(-4) 1	(-12) 5	0
(3) 8	2 9	14 2	(-9) 7	5
(11) 4	(10) 3	1 6	4 2	9
6	4	-3	-7	

Step -4

Select the cell where highest positive number in the non basic cell exist. In the present problem. The cell (3,1) has the highest +ve number II. now a loop is constructed starting from this cell through the basic cells and ends in the starting cell. A small value Q will be added and substracted alternatively from each all as shown in the following.

6 - θ 6	8 + θ 4	(-4) 1	(-12) 5
(3) 8	2 - θ 9	14 + θ 2	(-9) 7
(11) + θ 4	(10) 3	1 - θ 6	4 2

The minimum value of θ is min (6,8,2,14,1) = 1

(Note : A loop will be formed if it connect even no. of cells. here it is 6)

Step – 5

The new basic feasible solution and the net evaluation are presented in the following table.

5 6	9 4	(-4) 1	(-1) 5	0
(3) 8	1 9	15 2	(2) 7	5
1 4	(-1) 3	(-11) 6	4 2	-2
6	4	-3	4	

Step – 6

Select the cell where highest positive numbers in the non basic cell exist. In the present problem, the cell (3,1) has the highest +ve number II. Now a loop is constructed starting from this cell through the basic cells and ends in the starting cell. A small value Q will be added and substracted alternatively from each all as shown in the following.

5 – θ 6	9 + θ 4	(-4) 1	(-1) 5
(3) θ 8	1 - θ 9	15 2	(2) 7
4 4	3	6	4 2

→

4 6	10 4	1	5
1 8	9	15 2	7
1 4	3	6	4 2

Now the net evaluation for the table in prepare-

4 6	10 4	(-1) 1	(-1) 5	0
1 8	(-3) 9	15 2	(-1) 7	2
1 4	(-1) 3	(-8) 6	4 2	-2
6	4	0	4	

Since all the net evaluation are –ve, it is the optimal one. The optimum transportation cost = 4x6+10x4+1x8+15x2+1x4+4x2 = 114 Unit.

Degeneracy in transportation problem

If in a transportation problem (with on origin and 'n' destinations) the total numbers of positive basic variables < (m+n-I), then the transportation problem is a degenerate one.

It can develop in 2 ways :

(i) While determining the initial basic feasible solution following the 5 methods.

(ii) At some iteration stage used for getting the optimum solution.

How to remove/eliminate degeneracy?

A sample problem can be taken.

Prob.: obtain an optimum basic feasible solution to the following degenerate transportation problem.

7	3	4	2	
2	1	3	3	Available
3	4	6	5	
4	1	5	10	

Demand

Following the North-west corner rule an initial assignment in made as shown below.

2			2
2	1		3
		5	5
4	1	5	

In this case, the total no. of basic cells = 4, m+n-I = 5

Now 4 < 5

Hence the basic solution degenerate. in order to remove the degeneracy we require only one +ve variable. this is done in the following way.

Step – 1

A small +ve quantity € is selected and is allowed in the cell (2,3) as shown in the following table.

2 7	3	4	2
2 2	1 1	6 3	3+E
3	4	5 6	5
4	1	5+E	

It is seen that even after this, the basic cell do not form a loop i.e., the augmented solution remains basic (A feasible solution which does not form a loop is called basic)

Step – 2

The net evaluation is now computed and tested for optimality.

Since all the net evaluation for the non basic variables are not –ve, the initial solution is not optimum. Now the non basic cell (1,3) must enter the basic (because it has highest the value)

2 - ϴ 7	(3) 3	ϴ (4) 4	0
2 + ϴ 2	1 1	ϴ ≥0 3	-5
(2) 3	(0) 4	5 6	-2
uj 7	6	8	

The above integration is repeated.

2 - Θ 7	(3) 3	E + Θ 4	0
2 2	1 1	(-4) 3	-5
(6) Θ 3	(4) 4	5 - Θ 6	2
7	6	4	

Now Θ = 2, so the values in the table are:

	(-6) 7	(-3) 3	2 4	0
	2 - Θ 2	1 1	Θ (2) 3	1
	2 + Θ 3	(-2) 4	3 – Θ	2
Uj	1	0	4	

(Taking Θ = 2) →

	(-6) 7	(-1) 3	2 4	0	
	(-2) 2	1 1	2 3	-1	←optimal solution
	4 3	(0) 4	1 6	2	
Uj	1	2	4		

& optimal transportation cost = 2×4+1×1+2×3+4×3+1×6 = 33 unit.

CHAPTER-8 : ASSIGNMENT & ROUTING PROBLEM

1. © The assignment problem

The assignment problem is a special case of the transportation problem is to assign a number of origins to equal number of destinations at a minimum cost.

Ex: Let there are n jobs and n machines in a shop, and when ith job is assignment is to be made in such a way that the total cost would be minimum.

2. © Assignment Algorithm

Step-1: Determine the effective matrix

Subtract the minimum element of each row of the given cost matrix from all the elements of the row. Examine if there is at least are zero in each row and in each column. If it is so, stop here, otherwise subtract the minimum element of each column from all the elements of the column. The resulting matrix is starting effective matrix.

Step-2: Assign the zeros

(a) Examine the rows of the current effective matrix successively until a row with exactly one unmarked zero is found. Matrix this zero (by encircling it) indicating that an assignment will be made there. Cross mark all other zeros lying in the column of above encircled zero. The marked cell will not be considered for future assignment.

Continue in this manner until all the rows have been taken care of.

(b) Examine the column successively until a column with exactly one unmarked zero is found. Encircle that zero as an assignment will be made there. Cross mark all other zeros in the row of the above encircled zero. Continue in this way until all the column have been taken care of.

Step-3: Check for optimality

Repeat step 2 (a) and (b) successively until one of the following occurs.

(i) There is no row and no column without assignment. In such a case, the current assignment is optimal.

(ii) There may be some row or column without an assignment. In such case, the current solution is not optimal. Proceed on the next step-4.

Step-4: Draw minimum no. of lines

Draw minimum no. of lines crossing all zeros, as follows:

(a) Mark (□) the row in which assignment has not been made.
(b) Mark (□) the column (not already marked) which have zeros in the marked rows.
(c) Mark (□) rows (not already marked) which have assignments in the marked column.
(d) Repeat (b) & (c) until no more marking is possible.
(e) Draw lines through all unmarked rows and through all marked columns. Let the no. of these lines be N and let the order of effectiveness matrix be n. Then
(f) $N = n$:- current assignment is optimal.
(g) $N<n$:- current assignment is not optimal.

Step-5: Get the new effective matrix

Examine the elements that do not have a line through them. Select the smallest of those element and substract the same from all the elements that do not have a line through them. Add this smallest element to every element that lies at the intersection of the two (horizontal & vertical) lines. The resulting matrix is the new effectiveness matrix.

Step-6: Go to step-2 and repeat the procedure until the optimal assignment is reached.

Note: The above iterative method of determining an assignment schedule is known as Hungarian Assignment method.

The above algorithm can be best understood if the following 2 problems are solved.

problem-1: Assign 5 jobs to 5 persons. The assignment costs are given in the following table.

Persons	Job 1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule.

Step-1: In each row select the minimum cost and substract that from each. Similar operation is also done for each column and the resulting effective matrix:-

$$\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & 1 & 5 & 0 \\
B & 0 & 9 & 5 & 5 & 4 \\
C & 1 & 6 & 7 & 0 & 4 \\
D & 4 & 3 & 1 & 0 & 3 \\
E & 4 & 0 & 3 & 4 & 0 \\
\hline
\end{array}
\rightarrow
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & 0 & 5 & 0 \\
B & 0 & 9 & 4 & 5 & 4 \\
C & 1 & 6 & 6 & 0 & 4 \\
D & 4 & 3 & 0 & 0 & 3 \\
E & 4 & 0 & 2 & 4 & 0 \\
\hline
\end{array}$$

Step-2: Draw minimum number of horizontal and vertical lines so as to cover all the zeros.

$$\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
A & 7 & 3 & 0 & 5 & 0 \\
B & 0 & 9 & 4 & 5 & 4 \\
C & 1 & 6 & 6 & 0 & 4 \\
D & 4 & 3 & 0 & 0 & 3 \\
E & 4 & 0 & 2 & 4 & 0 \\
\hline
\end{array}$$

In this case, the no. of lines = 5 = order of matrix. So an optimum assignment has been attained.

Step-3: To determine the optimum assignment, only zeros are considered as shown below.

	1	2	3	4	5
A					
B					
C					
D					
E					

Now the optimum assignment :-

A – 5

B – 1

C – 4

D – 3

E – 2

& the minimum assignment cost = 1+0+2+1+5 = 9 unit.

Problem-2: A marketing manager has 5 salesmen and 5 sales district. Considering the capabilities of the salesmen and the nature of district, the marketing manager estimates the sales per month (is hundred rupees) for each salesman in each district would be as follows:

	D_1	D_2	D_3	D_4	D_5
S_1	32	38	40	28	40
S_2	40	24	28	21	36
S_3	41	27	33	30	37
S_4	22	38	41	36	36
S_5	29	33	40	35	39

Solution: Since max z = - min (-z), we first convert the given maximum problem into a minimization problem by making all the profits negative and the modified values will basic

Table - 1

-32	-38	-40	-28	-40
-40	-24	-28	-21	-36
-41	-27	-33	-30	-37
-22	-38	-41	-36	-36
-29	-33	-40	-35	-39

Now substracting the minimum element of each row from the elements of that row and then substracting minimum element of each column from the element of that column, the resulting table would be tab.3.

Table 2

8	2	0	12	0
0	16	12	19	4
0	14	8	11	4
19	3	0	5	5
11	7	0	5	1

Table 3

8	0	0	7	0
0	14	12	14	4
0	12	8	6	4
19	1	0	0	5
11	5	0	0	1

Then minimum no. of lines are drawn covering all zeros & shown in table-4 :-

8	0	0	7	0
0	14	12	14	4
0	12	8	6	4
19	1	0	0	5
11	5	0	0	1

Since the no. of lines is less than order of the cost matrix (=5) we should select the smallest element from the cost matrix not covered by the lines. The element is 4. Now substracting the element from the surviving element of cost matrix and adding the same to the element lying at the intersection of 2 lines.

Then minimum no. of lines are drawn as described above and the resulting table 5 becomes:

12	0	0	7	0
0	10	8	10	0
0	8	4	2	0
23	1	0	0	5
15	5	0	0	1

Since the no. of lines = order of cost matrix an optimum solution has been attained. In order to determine the optimum assignment only zeros are considered & the assignment is done as described earlier.

S1- D

S2 – D1

S3 – D5

S4 – D3

S5 – D4

© Routing Problem

Network scheduling is a technique for the planning and scheduling of large projects. It has successfully been applied in transportation and communication problem.

A typical network problem consists of finding a route from one node (origin) to another (destination) between which alternate paths are available at various stages of journey. The problem is to select the route that yields minimum cost. A number of different constraints may be placed on acceptable route. For example, not returning to the node already passed through or passing through every node once and only once. Problems of such type are called routing problems.

Although a large varieties of problems other than routing one may be developed in connection with the construction and utilization of networks, here we shall consider only the special type of routing problem, that occurs most frequently in O.R. – the travelling salesman problem.

Travelling salesman problem

Suppose a salesman has to visit 'n' cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The objective is to select the sequence in which the cities are visited in such a way that his total travelling time (or cost) is minimized. Clearly starting from a given city, the salesman will have a total of (n-1)! Different sequences (possible round trips). Further, since the optimal solution remains independent of the selection of starting point.

Formulation of travelling salesman problem as an Assignment Problem.

This can be best explained with the help of a sample problem.

Problem-3: A salesman wants to visit cities A,B,C,D and E. He does not want to visit any city twice before completing his tour of all the cities and wishes to return to the point of starting journey. Cost of going from one city to another (in Rs) is shown below. Find the least cost route.

	A	B	C	D	E
A	0	2	5	7	1
B	6	0	3	8	2
C	8	7	0	4	7
D	12	4	6	0	5
E	1	3	2	8	0

Solution:

As going from A → A, B → B, etc is not allowed, assign a large penalty (cost of journey) for these cells in the cost matrix. And the problem will be converted into an assignment problem. following this rule the above cost matrix would be:

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	0
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	0	7	∞

	A	B	C	D	E
A					
B					
C					
D					
E					

This table provides an optimum solution to the assignment problem but not to the travelling salesman problem as it gives A-E, E-A, B-C, C-D, D-B as the solution which means that the salesman should go from A to E and then come back to A without visiting B,C &D. This violats the additional constraints that the salesman is not to visit any city twice before completing his tour of all the cities.

Therefore, we have to try for the next best solution that also satisfies this additional constraints. The next minimum (non-zero) element in the matrix is 1. So we shall try to bring 1 along with zero. However, this 1 occurs in 3 different cells. We shall consider all the 3 cases until an acceptable solution is obtained.

Case-I

We make unit assignment in cell (A,B) instead of zero assignment in cell (A,E)

	A	B	C	D	E
A					
B					
C					
D					
E					

A-B-C-D-E-A

Feasible assignment

Cost = 2+3+4+5+1=15

Case-II

We can make unit assignment in cell (D,C) instead of zero assignment in cell (D,B). As a result we have to make assignment in cell (B,E) instead of cell (B,C) and in cell (A,B) instead of cell (A,E) and the assignment table :-

	A	B	C	D	E
A					
B					
C					
D					
E					

And the assignment becomes:

A-B-E-A / not feasible

C-D-C/ not feasible

Case-III: Unit assignment in cell (D,E)

	A	B	C	D	E
A					
B					
C					
D					
E					

And the assignment becomes:

A-B-C-D-E-A with cost = 15

So the least cost is Rs 15/- and the optimum assignment/routing is A-B-C-D-E-A.

CHAPTER-9: QUEUEING THEORY

1. © Introduction

A flow of customers from infinite/finite population towards the service facility forms a queue (waiting line) an account of lack of capability to serve them all at a time. The queue may be

(i) Persons waiting at doctor's clinic.
(ii) Persons waiting at doctor's railway booking office/counter.
(iii) m/cs waiting to be required.
(iv) Ships in the harbor waiting to be unloaded etc.

In the absence of a perfect balance between the service facility and the customers, waiting is required either of the service facility or of the customers arrival.

By the term customers we mean the arriving unit that requires some service to be performed. The customers may be of persons, machines, vehicles, parts, etc. Queue (waiting line) stands for a number of customers waiting to be serviced. The queue does not include the customer being served. The process/system that performs the services to the customer is termed by service channel or service facility.

The following figure (fig 9.1) shows the major constituents of a queuing system.

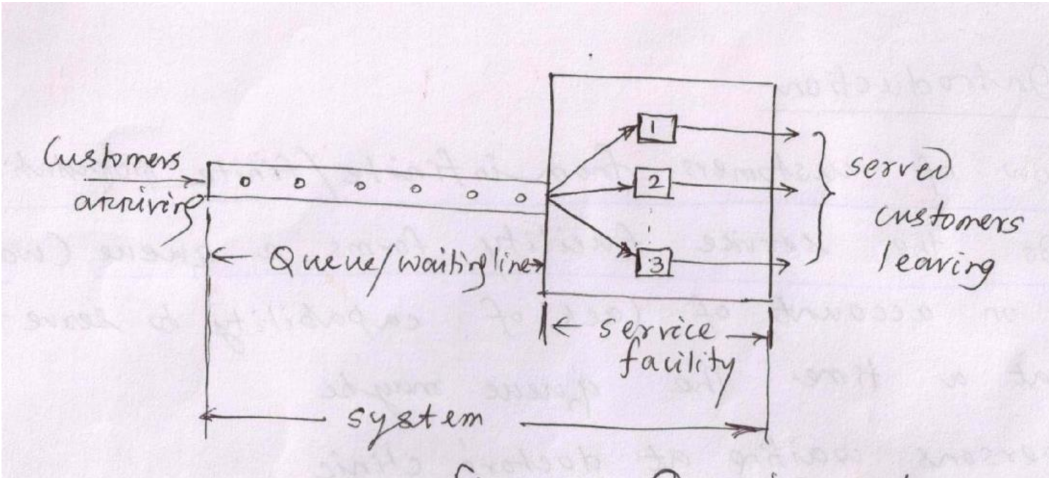


Fig 9.1 :- Queuing system

©

Classification

(I) Single server :- Registration in post office.

(II) Multiserver :- Railway reservation.

© Simple Queuing model

Single – channel Poisson's arrival with Exponential service, infinite population model.

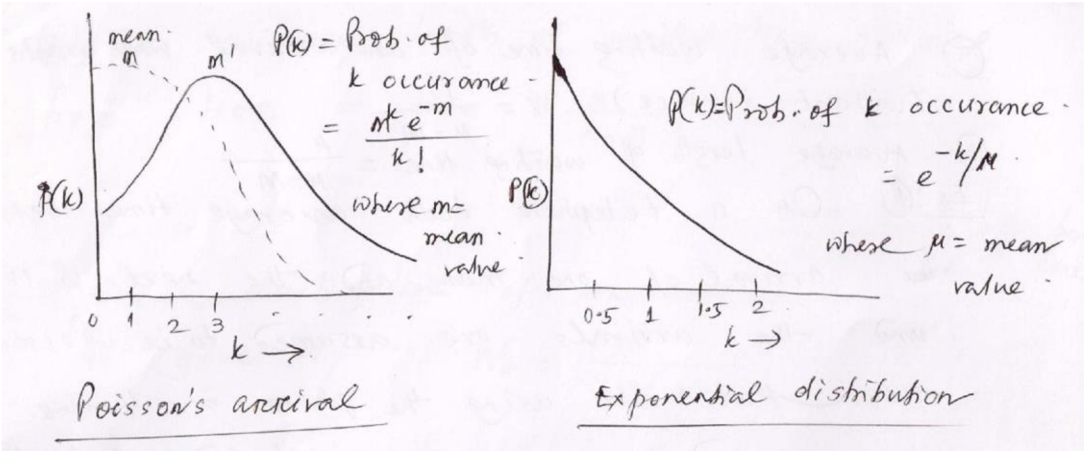
Assumptions

(i) Only one type of queue discipline (i.e., first come, first served).

(ii) Steady state condition exist (i.e., no. of customers in the queue remains same).

(iii) The no. of customers in the queue & waiting time experienced in a queue is independent of time (:- does not vary with time).

(iv) Arrival rate follows – Poisson's distribution & service follow – Exponential distribution.



Notations:

n = no. of customers in the system (both waiting + service).

Pn = Probability that there are n customers in the system.

M = mean arrival rate (average no. of customers arriving per unit time).

μ = mean service rate (average no. of customers being served per unit time).

$\frac{M}{\mu}$ = P = traffic density

Essential formula

(1) $P_n = (\frac{M}{\mu})^n [1 - \frac{M}{\mu}]$ ---(1) Eq(1) holds good when $\mu > M$.

(2) Average no. of units in the system (including the one under service).

$L = \frac{M}{\mu - M}$

(3) Average queue length (mean nos in the queue) $(L_q) = (\frac{M}{\mu}) \times (\frac{M}{\mu - M})$.

(4) Mean waiting time of an arrival $Wm = \frac{M}{M(\mu - M)} = (\frac{M}{\mu}) \times (\frac{1}{\mu - M})$.

(5) Average waiting time of an arrival who waits (without service)

$W = \frac{1}{\mu - M}$

(6) Average length of waiting line = $\frac{\mu}{\mu - M}$

Ex:

(1) On a telephone both average time between the arrival of one man and the next is 10 mins. and the arrivals are assumed to be poisson. The mean time of using the phone is 4 mins. and is assumed exponentially distributed. Calculate

i. The probability that a man has to wait after he arrives.
ii. The average length of waiting lines forming from time to time.
iii. By how much the flow of arrivals should increase to justify the installation of a second telephone.

Assume that second booth can be provided only a person has to wait minimum for 4 mins for the phone.

Solution:

In 10 mins there is one arrival therefore, in 1 min → $\frac{1}{10}$arrival

So, M = $\frac{1}{10}$ = 0.1

Service time = 4 mins :- service per unit time = $\frac{1}{4}$ = 0.25

So, μ = 0.25

Hence μ > M

(1) Probability that one has to wait = 1 − P_0
P_0 = $(1 - \frac{M}{\mu})$ So, 1 − P_0 = 1- $[1- \frac{M}{\mu}]$ = $\frac{M}{\mu}$ = $\frac{0.1}{0.25}$ = 0.4

(2) The average length of waiting lines forming from time to time
$= \frac{M}{\mu - M} = \frac{0.25}{0.25 - 0.1} = \frac{0.25}{0.15}$ = 1.66 per hour.

(3) Mean waiting time of an arrival is given by: Wm = $\frac{M}{M(\mu - M)}$

$= 4 = \frac{M^1}{0.25(0.25 - M^1)} = M^1$ = 0.125 per min

i.e., 0.1 per min to 0.125 per min
i.e., 6 per hour to 8 per hour to install 2nd booth.

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